

05/14/2009

**STUDY SHEET for TEST #1 (Friday, May 15)**

The test will cover:

- Ch. 1; Sections: 1.1, 1.2, 1.3, 1.5, 1.6
- Ch. 2; Sections 2.1, 2.2, 2.3

The best way to prepare to the exam is to read the book and to do the homework exercises. Please, take time to go over the material.

Here is what you have to have an idea about:

1. Four ways to represent a function. The vertical line test. Piecewise defined functions. Symmetry. Odd and even functions. Increasing and decreasing functions. (1.1)
2. Mathematical models: a catalog of essential functions: Linear functions, Polynomials, Power functions, Rational functions, Algebraic functions, Trigonometric functions, Exponential functions, Logarithmic functions, Transcendental functions. (1.2)
3. Transformation of functions. Composition of functions. (1.3)
4. Exponential functions and their properties. Applications. The number  $e$ . (1.5)
5. Inverse functions. One-to-one functions. Horizontal line test. (1.6)
6. Logarithmic functions and their properties. Natural logarithm. Change of a base formula. (1.6)
7. Inverse trigonometric functions. (1.6)
8. The tangent and velocity problems. (2.1)
9. Limit of a function. One-sided limits. Infinite limits. Vertical asymptotes. (2.2)
10. Limit laws. Direct substitution property. Condition for two-sided limit existence in terms of one-sided limits.

Below is the list of typical problems in the most general from which you can expect on the test:

1. Sketch a rough graph of the market value of a new car as a function of time for period of 20 years. Assume the car is well maintained.
2. Given a function, find its domain.
3. Sketch the graph of a piecewise defined function.
4. A Norman window has the shape of a rectangle surmounted by a semicircle. If a perimeter of the window is 30 ft, express the area  $A$  of the window as a function of the width  $x$  of the window.
5. Determine whether a function is even, odd or neither.
6. Classify given functions as a linear, power, root, polynomial, rational, algebraic, trigonometric, exponential, logarithmic, or transcendental function.
7. The manager of a furniture factory finds that it costs \$2200 to manufacture 100 chairs in one day and \$4800 to produce 300 chairs in one day.
  - a) Express cost as a function of the number of chairs produced, assuming that it is linear.

- b) What is the slope of the graph and what does it represent?  
 c) What is y-intercept of the graph and what does it represent?
8. For a given function  $f(x)$ , write equation of the graph that is obtain from  $f$  by shifting 3 units to the right and reflecting about the x-axis.
9. Graph a function by hand, not by pointing points, but by starting with the graph of one of the standard functions and then applying the appropriate transformations.  
**Do not use a calculator.**
10. Given functions  $f(x)$  and  $g(x)$ , find  $f \circ f$ ,  $f \circ g$ ,  $g \circ f$ , and  $g \circ g$ .
11. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.  
 a) Express the radius  $r$  of this circle as a function of the time  $t$  (in seconds).  
 b) If  $A$  is the area of this circle a function of the radius, find  $A \circ r$  and interpret it.
12. Suppose  $f$  and  $g$  are even functions. What can you say about  $f + g$  and  $fg$ ? What is  $f$  and  $g$  are both odd?
13. Graph the given exponential functions on a common screen. How are these graphs related?
14. Make a rough sketch of the function  $f(x) = \frac{1}{2}(3 - e^{2x})$ . **Do not use a calculator.**
15. Given a half-life period of a certain radioactive element and the initial amount, compute how much will be left after 1000 years?
16. A function is given by a table of values, a graph, a formula, or by a verbal description. Determine whether it is one-to-one.
17. Given graph of a function  $f$ , sketch the graph of its inverse  $f^{-1}$ .
18. For a given one-to-one function, find a formula for its inverse.
19. Evaluate logarithmic expressions.
20. Solve logarithmic and exponential equations.
21. Describe the inverse function of  $\cos x$ . What is its domain? What is its range?
22. Find the exact value of the expression  $\tan(\sec^{-1}4)$ .
23. Find the slope of a tangent line to the graph of a given function at a given point by limiting the slopes of the secant lines. Write the equation of this tangent line.
24. If a ball is thrown into the air with a given velocity and the formula for its height is given,  
 a) Find the average velocity for a certain time interval.  
 b) Estimate the instantaneous velocity at a given time.
25. Use the graph of a function to determine  $\lim_{x \rightarrow a} f(x)$  at the indicate value of  $a$ , if it exists.
26. Sketch the graph of the function  $f$  and evaluate  $\lim_{x \rightarrow a} f(x)$ , if it exists, for a given value of  $a = 3$ .
27. Find the indicated limit, if it exists.
28. Use the graph of a function to find  $\lim_{x \rightarrow a^-} f(x)$ ,  $\lim_{x \rightarrow a^+} f(x)$ , and  $\lim_{x \rightarrow a} f(x)$  at the indicated value of  $a$ , if the limit exists.