05/21/2009 **STUDY SHEET for TEST #2 (Friday, May 22)**

The test will cover:
- Ch. 2: Sections 2.3 (Squeeze Theorem), 2.4, 2.5, 2.6, 2.7, 2.8
-Ch. 3: Sections 3.1, 3.2, 3.3, 3.4

The best way to prepare to the exam is to read the book and to do the homework exercises. Please, take time to go over the material.

Here is what you have to have an idea about:

1. The squeeze theorem and its applications. (2.3)
2. The $\epsilon - \delta$ definition of limit. (2.4)
3. The $\epsilon - \delta$ definition of left-hand limit. The $\epsilon - \delta$ definition of right-hand limit. (2.4)
4. Infinite limits. (2.4)
5. Continuity. Definition of a continuous function at a point, continuous from the right, continuous from the left, continuous on an interval. Discontinuity. (2.5)
6. Properties of continuous functions. Theorems 4, 5, 7, 8, and 9 from the textbook. (2.5)
7. The intermediate value theorem and its applications. (2.5)
8. Limits at infinity (positive, negative), $\epsilon - \delta$ definition. (2.6)
9. Horizontal asymptotes. (2.6)
10. Infinite limits at infinity. (2.6)
11. The tangent line definition. Derivatives and rates of change. Applications. (2.7)
12. The derivative as a function. (2.8)
13. Notations for differentiation operators. Definition of differentiable function at a number, on an open interval. Differentiability and continuity. (2.8)
14. How can a function fail to be differentiable? (2.8)
15. Higher derivatives. (2.8)
16. Derivatives of polynomial and exponential functions. (3.1)
17. New derivatives from old. The constant multiple rule, the sum rule, the difference rule. (3.1)
18. Definition of the number *e*. The limit$\lim_{x\to 0} \frac{e^x - 1}{x}$. (3.1)
19. The product and quotient rules. (3.2)
20. Derivatives of trigonometric functions. The limits $\lim_{x\to 0} \frac{\sin x}{x}$ and $\lim_{x\to 0} \frac{\cos x - 1}{x}$ (3.3)
21. The chain rule. The derivative of an exponential function. (3.4)

Below is the list of typical problems in the most general from which you can expect on the test:

1. Show that $\lim_{x\to 0}(\sqrt{x^3+x^2}\cdot sin\frac{\pi}{x}) = 0$.
2. Prove that $\lim_{x\to 3}(2x+1) = 7$ by using $\epsilon - \delta$ definition of limit.
3. How close to -3 do we have to take *x* so that $\frac{1}{(x+3)^4} > 1{,}000{,}000$. Prove that that $\lim_{x\to -3}\frac{1}{(x+3)^4} = \infty$.
4. Graph of a function is given. Determine the values of *x*, if any, at which the function is discontinuous. At each point of discontinuity, explain why the function is discontinuous at this point. At which of these points is *f* continuous from the right, from the left, or neither.
5. Determine the values of *x*, if any, at which each function is continuous /discontinuous:
   a) $f(x) = \dfrac{8x}{(x-1)(x-2)}$
   b) $f(x) = \begin{cases} x+1 & if\ x \le 1 \\ 3x-1 & if\ x > 1 \end{cases}$
6. Use the definition of continuity and the properties of limits to show that the given function is continuous on the given interval.
7. Use properties of continuous functions (theorems 4, 5, 7, 8, and 9 from the textbook) to explain why the given function is continuous at every number in its domain. State the domain.
8. Use continuity to evaluate the limit:
   a) $\lim_{x\to 4}\frac{5+\sqrt{x}}{\sqrt{5+x}}$
   b) $\lim_{x\to \pi}\sin(x+\sin(x))$
9. Use the intermediate value theorem to show that there is a root of the given equation in the specific interval.
10. For what values of *x* is *g* continuous?
$$g(x) = \begin{cases} 0 & if\ x\ is\ rational \\ x & if\ x\ is\ irrational \end{cases}$$
11. How many horizontal asymptotes can the graph of $y = f(x)$ have? How many vertical? Sketch the graphs to illustrate the possibilities.
12. Sketch the graph of a function that satisfies all of the given conditions.
13. Find the limits.
    a) $\lim_{x\to -\infty}\frac{3x^2-5x+3}{\sqrt{5x^4+x}}$
    b) $\lim_{x\to \infty}\frac{3x^2-5x+3}{x^2-x^4}$
    c) $\lim_{x\to \infty} cot^{-1}(x^3-x^2)$
    d) $\lim_{x\to \infty}(x^{100}e^{-x})$
    e) $\lim_{x\to \infty}\cos(x)$
    f) $\lim_{x\to -\infty}(x^5-x^6)$
    g) $\lim_{x\to -\infty}(\sqrt{x^2+x+1}+x)$
14. Find the horizontal and vertical asymptotes of a given function. Check your answers!

15. Find the limits as $x \to \infty$ and as $x \to -\infty$. Use the given information, together with intercepts, to give a rough sketch of the graph of a given function.
16. Find the slope and the equation of the tangent line to the graph of the function $f(x) = x^2 + 3x$ at point (1, 4).
17. Find the slope and the equation of the tangent line to the graph of the function $f(x) = \frac{1}{\sqrt{x}}$ at point (4, 1/2). Find the slope at a point *a*.
18. If a ball is thrown into the air with velocity of 40 ft/s, its height (in feet) after *t* seconds is given by $H(t) = 10t - 1.86t^2$. Find the velocity when *t* = 2. Find the velocity when *t* = *a*.
19. Find the derivative $f'(a)$ of the function *f* at a point *a* by using the definition of derivative.
    a) $f(x) = 365$
    b) $f(x) = \sqrt{x-3}$
    c) $f(x) = 2x^2 - 3x$
20. Given the graph of a function, make a sketch of the graph of its derivative.
21. Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative. Sketch the graphs of *f* and f' by using the same set of axes.
22. For the given graph of a function *f*, state, with reasons, the numbers at which *f* is not differentiable.
23. Use the definition of derivative to find $f'$and f''.
24. Differentiate a function. Do not simplify.
    a) $f(x) = 3x^5 - 4x + e^x$
    b) $f(x) = (x-5)^2(x-2)^5$
    c) $f(x) = \left(\frac{2x^3-2x}{x \sin x}\right)^8$
    d) $f(x) = \tan(\sqrt[4]{x^2 e^x})$

25. Find an equation of the tangent line to the graph of a given function at a given point.
26. Find the points on the graph of a function at which the tangent line to the graph is horizontal.
27. The curve $y = \frac{1}{1+x^2}$ is called a witch of Maria Agnesi. Find an equation of the tangent line to this curve at the point (1, 1/2).
28. Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$.
29. Find $\lim_{x \to 0} \frac{\tan 6x}{\sin 2x}$.
30. A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by ± 0.35. In view of these data, the brightness of Delta Cephei at time *t*, where *t* is measured in days, has been modeled by the function $B(t) = 4.0 + 0.35 \sin(\frac{2\pi t}{5.4})$. Find the rate of change of brightness after *t* days.