Laboratory 1

Experimental Error and Spreadsheets

Name ______________________ Partners ______________________

Lab Date & Time ______________________ Instructor ______________________

I. Objectives

After completing this lab, you should be able to:

A. Explain what experimental error is and what it is not.

B. Explain what precision is and why it almost always contributes to experimental error.

C. Propagate errors from measured to calculated quantities.

D. Explain the difference between systematic and random errors.

E. Perform simple spreadsheet operations, including

  a. data entry,

  b. relative addressing,

  c. absolute addressing,

  d. calculating simple formulas,

  e. calculating the mean and standard deviation.

II. Overview

Many words have different meaning in the context of physics than that of every-day life. For example, we say that Congress has the power to declare war or that Superman has the power of flight, but in physics power is the energy consumed (or generated) per unit time. We say that work is something done for pay, but in physics work is force exerted over a distance. Likewise, error in physics, especially experimental error, is not the same as error in other contexts; it does not refer to an error that you make out of carelessness or confusion, but to *uncertainty that* *is inherent in the experiment* even if it is performed carefully by someone with experience. For that reason, *you should never refer to "human error" in a lab report;* not only is it vague, it usually means carelessness. You may, however, refer to something more specific, such as the limits of your eyesight or hearing that affect the outcome of the experiment.

For example, in Lab 12 you will need to listen for the difference in loudness between when the tube is in front of the speaker and when it away from the speaker to identify the resonant frequency. At the resonant frequency, the difference is largest. The problem is that for frequencies near the resonant frequency, the difference is almost as large as at the resonant frequency. Because your perception of sound and your memory of that perception are not perfect, you will not be entirely certain where the resonant frequency is; you will only know the resonant frequency to within one or two Hertz. This is not "human error"; it is a lack of precision inherent in the instrument, which happens to be human hearing.

Limited precision is one of the most common sources of experimental error, because it is inherent in *every* quantitative physical measurement. For example, the smallest markings on the metric side of a ruler are millimeters (which is why many rulers have "mm" printed on that side, even though the numbered marks are for centimeters). As a result, direct measurements by the ruler are only meaningful to about the nearest millimeter; there is no way the ruler could distinguish 0.2157 m from 0.2158 m.

It often happens that an experiment that is carefully repeated several times by the same experimenter using the same equipment nevertheless yields a (narrow) range of resulting measurements. This is called *random error*, and it is due to factors that are practically impossible to observe. For instance, in Activity 2-1 of Lab 5, you will push a small metal ball into a spring-loaded projectile launcher until you hear the second "click", then launch it into the air and measure how high the ball goes above the end of the launcher. The difficulty of seeing the ball reach the top of its trajectory at the same time you see the meter stick will limit the precision of your measurement to about one centimeter, but the variation in height you observe may be closer to five centimeters—even though you follow exactly the same procedure and the equipment appears to be returned to an identical state before each launch. Presumably the spring sometimes buckles or twists slightly, depending on factors such as how quickly the spring is compressed and how uniform the pressure is on the cup at the end of the spring. Regardless of what is happening, whenever random error is present, it is necessary to repeat the experiment several times and average the measurements, being very careful to perform the setup each time in exactly the same way.

Contrasting with random errors are *systematic errors*. Whereas random errors result in measurements that are sometimes too large and sometimes too small, systematic errors result in measurements that are either consistently too large or consistently too small.

Systematic errors can be due to miscalibrated instruments. For example, your car's speedometer determines the car's speed from the rotational speed of the wheels and the diameter of the tires. If, however, you install tires with a larger radius without adjusting the speedometer, the speedometer will *systematically* (that is, consistently) underestimate the speed, and if you install tires with a smaller radius without adjusting the speedometer, the speedometer will systematically overestimate the speed.

Systematic errors can also arise from idealization. A good example is the simple pendulum in Lab 11. The equations used in Lab 11 assume that the maximum angle is small, all the mass is concentrated at the center of the metal (or wood) ball, and the force of wind resistance is completely negligible compared to the weight of the ball and the tension in the string. The physical pendulum in Lab 11 will show you what happens if the mass is not all concentrated in a single point, but if either the angle or wind resistance is not tiny, the acceleration due to gravity will be systematically underestimated. (However, if you follow the directions in Lab 11, random errors associated with timing the swings of the pendulum will be much more important than the small systematic errors.)

III. Propagation of Errors

It is often easy enough to understand the experimental errors in quantities that are directly measured, but the real purpose of the experiment is to determine some function of your measurement(s). In principle this is a topic that requires a good knowledge of calculus and statistics, but only a few results are necessary for PHY 202. We assume all errors are small and independent of each other.

In the remainder of this section, the measured quantity will be called x (if there are multiple measured quantities, they will be x_1, x_2, x_3, etc.), the calculated quantity will be called y, small delta (δ) means "the uncertainty in …". To be consistent with your lectures, capital delta (Δ) will continue to mean "the increment in …." Other letters represent constants. Also, remember that the uncertainty is always positive, even though the measurement may be too large or too small. If the calculated uncertainty is negative, take the absolute value.

IV. Error Propagation for Linear Functions

$$\text{If } y = Ax + B, \text{ then } \delta y = A\delta x. \qquad (1)$$

Example: Suppose in Lab 10 you measure the time required for a mass on the end of a spring to complete 20 full cycles and find it is $x = 11.70 \pm 0.03$ s, with the uncertainty in stopwatch measurements being something you will estimate more accurately for yourself later in this lab. What are the value and uncertainty for the time required to complete one full cycle?

Since $y = (1/20)\,x + 0$, $y = 0.585$ s; and $\delta y = (1/20)\,\delta x$, so $\delta y = 0.0015$ s. This could be reported as: $T = 0.5850 \pm 0.0015$ s.

V. Error Propagation for Power Laws

$$\text{If } y = cx^n\text{, then } \delta y = (ny / x)(\delta x). \qquad (2)$$

Note: n *does not have to be an integer, and it can be positive or negative.*

Example: Continuing with Lab 10, suppose you have measured the mass to be $m = 0.100$ kg (for now we will ignore the uncertainty in mass) and determined the period to be $T = 0.5850 \pm 0.0015$ s. The spring constant k is given by $k = (4\pi^2 \text{ m}) T^{-2}$. What are the value and uncertainty for the spring constant?

Comparing the formulas, we see $k \to y$, $T \to x$, $\delta T \to \delta x$, $(4\pi^2 \text{ m}) \to c$, and $-2 \to n$. Plugging in the values, $y = 11.536$ N/m, and $\delta y = 0.059$ N/m, so $k = 11.536 \pm 0.059$ N/m.

VI. Error Propagation for Functions of Multiple Variables

$$\text{If } y = f(x_1, x_2, x_3, \ldots)\text{, then } \delta y = [(\delta y_1)^2 + (\delta y_2)^2 + (\delta y_3)^2 + \ldots]^{1/2}, \qquad (3)$$

where δy_1 is the uncertainty if all the variables other than x_1 are held constant, δy_2 is the uncertainty if all the variables other than x_2 are held constant, etc.

Example: Staying with Lab 10, let us now consider not just the mass at the end of the spring, but the mass of the spring itself, which we will suppose has been measured to be 0.008 kg. How much of the spring's mass should be added to the mass at the end to find the effective mass in the formula for the spring constant, $k = (4\pi^2 m) T^{-2}$? Presumably it is between zero and half the mass of the spring, so $m = 0.102 \pm 0.002$ kg.

We can immediately evaluate the spring constant to find k = 11.767 N/m. Comparing the formulas, we see $k \to y$, $T \to x_1$, $\delta T \to \delta x_1$, $m \to x_2$, and $\delta m \to \delta x_2$. Holding the mass fixed, the spring constant is proportional to a power of the period, as in the previous example, with $(4\pi^2 \text{ m}) \to c$, and $-2 \to n$; the resulting part of the uncertainty is $\delta y_1 = 0.060$ N/m. On the other hand, holding the period fixed, the spring constant is a linear function of the mass, with $(4\pi^2 T^{-2}) \to A$ and $0 \to B$; the resulting part of the uncertainty is $\delta y_1 = 0.231$ N/m. The total uncertainty in the spring constant is then $\delta y = 0.238$ N/m, so the spring constant should probably be reported as $k = 11.77 \pm 0.24$ N/m.

VII. Error Propagation for Averages of Measurements

If y is the average of N measurements x_i performed the same way so they each have the same **uncertainty** δx, so that

$$y = N^{-1} (x_1 + x_2 + x_3 + \ldots), \text{ then } \delta y = N^{-1/2}\, \delta x. \qquad (4)$$

This is a special case of a function of multiple variables, but it deserves particular attention because random error is common and always requires averaging over repeated measurements. The error δx of a single measurement is almost never known beforehand; instead, a statistical parameter called the **standard deviation** σ of the set of measurements is found, and it is assumed that $\delta x \approx \sigma$.

VIII. Spreadsheets

Most students come to college already familiar with at least one word processor (usually Microsoft Word) and at least one kind of presentation software (usually Microsoft PowerPoint), but many students surprisingly have little to no experience with spreadsheet software (such as Microsoft Excel). Spreadsheets are useful not only for recording and displaying lists of numbers, but also for performing calculations on these lists; your instructors probably use spreadsheets to calculate grades. More significant for introductory physics is the fact that you can use a spreadsheet to perform any calculation you can perform with a scientific calculator. One of the goals of Lab 1 is to familiarize you with some of the basic features of spreadsheet software – particularly Microsoft Excel, because it is installed on almost every computer on campus and is available to students at *https://www.marshall.edu/it/office365/*. These instructions are for Microsoft Excel, but LibreOffice Calc (freely available at *https://www.libreoffice.org/*) and Google Sheets (*https://docs.google.com/spreadsheets/u/0/*) are very similar. Each of these has many free online sources of information, including tutorials and solutions to common problems from the community of the software's users.

The basic component of a spreadsheet is a cell, a rectangular box that presents text or a number. Cells are organized into rows and columns, and the rows and columns are organized into sheets. The leftmost column is column A, and columns increase alphabetically from left to right; after column Z comes column AA, followed by AB and AC, etc. The topmost row is row 1, and rows increase numerically from top to bottom. By default, there are three sheets, cleverly named Sheet1, Sheet2, and Sheet3; these can be accessed through tabs at the bottom of the user interface. Sheets can be added or copied if needed.

Each cell can be empty, or it can contain text, a number, or a formula. Text is what it sounds like; common examples include a row or column heading or a list of the names of students in a class. Numbers include integers (like –5), decimal numbers (like 3.14159), and numbers in scientific notation (like 1.6022E–19, which means 1.6022×10^{-19}), but they also include dates and times, which are specially formatted numbers. Excel will try to guess the intention of what you type in, which can lead to minor frustrations. If you type in "0.25", that is understood to be a number; if you type "1.0/4.0", that is stored as text; if you type "1/4", that will be stored as the text symbol "¼"; if you type "4/1", it is understood to be April 1 of the current year and is presented in the default format for a date; and if you type in "=1/4", it is stored as a formula that immediately evaluates to the number 0.25. All formulas start with an equals sign.

The arguments of functions are not usually constants typed in directly, but are instead the contents of another cell, specified by its address, or by the contents of a range of cells. The *address of a cell* follows the pattern Sheet1.A1, giving the sheet name, a period, the column, and the row, in that order. The sheet name and period can be omitted if you are only using one sheet. A rectangular range of cells can be addressed according to the pattern A1:Z10.

If you copy or autofill formulas containing these addresses, Excel makes assumptions about what you mean. For example, maybe column A contains the names of students (beginning with row 2, since row 1 probably contains a heading) and columns C through M contain grades for each of the 11 quizzes each student has taken. Column B is for each student's quiz average, with the lowest score dropped in each case. You would type in cell B2 the formula, "=(SUM(C2:M2)-MIN(C2:M2))/(COUNT(C2:M2)-1)", then "grab" the lower right-hand corner of cell B2 and "drag" it down; Excel would copy the formula to the other cells, changing the row numbers in the addresses to the row number holding the formula. These are called *relative addresses.* If you do not want Excel to change the addresses, put a dollar sign ("$") in front of the part you want to keep the same. For instance, $B2 will not change the column from B, B$2 will not change the row from 2, and B2 will not change the row or the column. These are called *absolute addresses.*

IX. Investigation 1: Error in the Area of a Photograph

A. Activity 1-1: Error in the Width of a Photograph

Use a ruler to measure the width of the photo of the fountain below.

Figure 1-1

Width of the photo to the nearest cm: w_{cm} = ______________ cm

What is the smallest division on the "inches" side of your ruler? Circle one.

1/32 inch　　　　1/16 inch　　　　1/8 inch

Width of the photo to the nearest marked fraction of an inch: w_{in} = ______________ in

Given the fact that 1 inch = 2.54 cm, convert the width in inches to cm, keeping 5 decimal places:

w'_{in} = ______________ cm

Find $|w'_{in} - w_{cm}|$ = ______________ cm

B. Activity 1-2: Error in the Height of a Photograph

Use a ruler to measure the height of the photo of the fountain on page 7.

Height of the photo to the nearest cm: h_{cm} = ______________ cm

Height of the photo to the nearest marked fraction of an inch: h_{in} = ____________ in

Given the fact that 1 inch = 2.54 cm, convert the height in inches to cm, keeping 5 decimal places: h'_{in} = ____________ cm

Find $|h'_{in} - h_{cm}|$ = ____________ cm

Question 1-1

You probably found that $|w'_{in} - w_{cm}| \neq 0$ and/or $|h'_{in} - h_{cm}| \neq 0$. How do you explain this?

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C. Activity 1-3: Propagating the Errors

What is the uncertainty in the measurement of the width in centimeters? δw_{cm} = ____________ cm

What is the uncertainty in the measurement of the height in centimeters? δh_{cm} = ____________ cm

Use w_{cm}, h_{cm}, δw_{cm}, and δh_{cm} to calculate the uncertainty δA in the area. Show your work.

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Question 1-2

How does δA compare with $|w'_{in} \times h'_{in} - w_{cm} \times h_{cm}|$?

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B. Activity 2-2: Spreadsheet Analysis

Open Excel and start a new, blank spreadsheet. In cell A1 type "Timings [s]". This will be your header. Highlight cells A1 through E1, then use the "Merge & Center" button so your header will span all your data. Then type the numbers you recorded in Activity 2-1 into cells A2 through E11.

In cell A14 type "Average = "; this is text, not a function, because it does not start with an "=". In cell B14 type "= AVERAGE(A$2:E$11)". This will show your average time. In cell C14 type "seconds".

Write the average time here: ____________ s

In cell A15 type "Standard Deviation = ". Grab the lower-right corner of cell B14 and drag down to B15, then edit cell B15 to read, "= STDEV(A$2:E$11)".

Write the standard deviation of the time here: ____________ s

Use the Overview to calculate the uncertainty in the average. Show your work.

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Uncertainty in the average = ____________ s

Question 1-3

Based on the average and the uncertainty in the average, is systematic error present in your timings? That is, do you have a meaningful tendency to be too slow or too fast to stop the stopwatch?

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XI. Final Questions

Question 1-4

Suppose the PASCO Motion Sensor is calibrated to give correct measurements of position at 20°C, but the actual temperature in the lab room is 22°C. The Motion Sensor works by timing how long a pulse of sound takes to go from the sensor to the cart and back to the sensor, and at 20°C, the speed of sound is 343.15 m/s, but at 22°C, the speed of sound is 344.31 m/s. What effect does this difference in the speed of sound make, and what kind of error is this?

Question 1-5

The PASCO Motion Sensor has a resolution of one millimeter, meaning position measurements may be too large or too small by one half millimeter (probably at random). What kind of error is this?

Question 1-6

The PASCO Motion Sensor has a resolution of one millimeter, meaning position measurements may be too large or too small by one half millimeter (probably at random). What kind of error is this?

Question 1-7

The PASCO Motion Sensor has a resolution of one millimeter in position measurements. It estimates the acceleration at time t according to the formula $a = (x_F - 2\,x_N + x_P)/(\Delta t)^2$, where $\Delta t = 0.05$ s is the time between successive position measurements, x_F is the position at time $t + \Delta t$ (future), x_N is the position at time t (now), and x_P is the position at time $t - \Delta t$ (past). What is the resolution of the Motion Sensor for acceleration measurements? Show your work.