

**Exam 1 MTH 122 Fall 2022 Total Pts:100 9/23/2021**

Name: \_\_\_\_\_

Total Received:

Show all work for full credit. Write all your solutions on the **blank papers**.

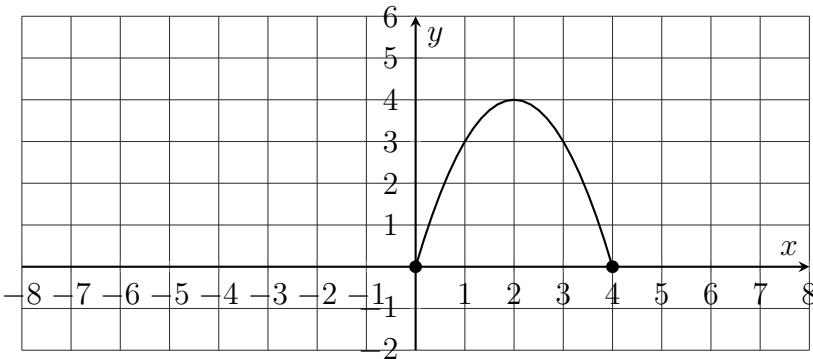
1. Complete the following table. (5 Pts)

|     | 0° | 30° | 45° | 60° | 90° |
|-----|----|-----|-----|-----|-----|
| sin |    |     |     |     |     |
| cos |    |     |     |     |     |
| tan |    |     |     |     |     |

2. Find an equation of the circle centered at (-2,1) and passing through (-4,2). (4 Pts)
3. Find the inverse function  $f^{-1}(x)$  of the function  $f(x) = 2x + 5$ . (4 Pts)
4. Find the inverse function  $f^{-1}(x)$  of the function  $f(x) = \frac{x+4}{x-3}$ . (5 Pts)

5. From the given graph of the function  $y = f(x)$ , make a hand-drawn graph of the following functions. (6 Pts)

(a)  $y = f(x) + 2$    (b)  $y = f(x + 2)$    (c)  $y = \frac{1}{2}f(x)$    (d)  $y = -f(x - 2)$



6. Find the functions  $f/g$ ,  $f \circ g$ , and  $g \circ f$  for the functions  $f(x) = x^2 + 2x - 5$ ,  $g(x) = \frac{2x+3}{x-2}$  (5 Pts)
7. Find the degree measures of two positive and two negative angles that are coterminal with angle  $125^\circ$ . (4 Pts)
8. Add the following:  $64^\circ 48'32'' + 39^\circ 38'28''$  and turn into decimal degree. (6 Pts)
9. Find arc length  $s$  if  $\alpha = 60^\circ$  and radius  $r = 20$  ft. (4 Pts)
10. Evaluate the following using calculator. (6 Pts)
  - (a)  $\sin(125^\circ)$
  - (b)  $\tan(4.7)$
  - (c)  $\cos(\frac{11\pi}{7})$
  - (d)  $\sec(-68^\circ)$ .
11. Find  $a$ ,  $c$ , and angle  $\alpha$  if  $\beta = 60^\circ$  and  $b = 7$  in for a right triangle. (6 Pts)
12. Convert the following. (6 Pts)
  - (a)  $285^\circ$ ,  $160^\circ$  (into radian)
  - (b)  $\frac{7\pi}{5}$ ,  $\frac{11\pi}{7}$  (into degree)

13. Name the quadrant containing the terminal side of  $\alpha$ . (4 Pts)  
(a)  $\sin \alpha > 0$  and  $\tan \alpha < 0$   
(b)  $\cos \alpha < 0$  and  $\cot \alpha > 0$
14. Evaluate each expression (in degree). (4 Pts)  
(a)  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$       (b)  $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
15. Find the exact value of each of the other five trigonometric functions for the angle  $\theta$  given that  $\cos \theta = -\frac{3}{4}$  and  $\theta$  is in the III quadrant. (6 Pts)
16. The angle of elevation to the top of a building in New York is found to be  $9^\circ$  from the ground at a distance of 1 mile from the base of the building. Find the height of the building (in ft).  
(Draw a diagram.) (5 Pts)
17. A router bit makes 45,000 rev/min. What is the linear velocity (in miles per hour) of the outside edge of a bit that cuts a 1-in.-wide path? (1 mile=5280 ft) (5 Pts)
18. Sketch the following angles in their standard positions and then find the reference angle:  
 $\frac{7\pi}{4}, \frac{4\pi}{3}, 210^\circ, 330^\circ, -120^\circ, \frac{7\pi}{6}$ .  
Use reference angle and table in number 1 to find the exact value of each trigonometric functions  
(No Calculator). (15 Pts)  
(a)  $\sin\left(\frac{7\pi}{4}\right)$     (b)  $\cos\left(\frac{4\pi}{3}\right)$     (c)  $\tan(210^\circ)$     (d)  $\cot(330^\circ)$     (e)  $\csc(-120^\circ)$     (f)  $\sec\left(\frac{7\pi}{6}\right)$

$$1. \quad \begin{array}{cccccc} 0^\circ & 30^\circ & 45^\circ & 60^\circ & 90^\circ \\ \frac{\sqrt{0}}{2} & \frac{\sqrt{1}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{4}}{2} \end{array}$$

Isha Gupta

$$\sin \quad 0 \quad \frac{1}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{\sqrt{3}}{2} \quad 1$$

$$\cos \quad 1 \quad \frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{1}{2} \quad 0$$

$$\tan \quad 0 \quad \frac{1}{\sqrt{3}} \quad 1 \quad \sqrt{3} \quad \text{undefined}$$

$$2. \quad (x-h)^2 + (y-k)^2 = r^2 \quad (-2, 1) \\ (x+2)^2 + (y-1)^2 = r^2 \quad (-4, 2)$$

$$r = \sqrt{(-2 - (-4))^2 + (1 - 2)^2}$$

$$\sqrt{(-2+4)^2 + (-1)^2}$$

$$\sqrt{(2)^2 + (-1)^2}$$

$$\sqrt{4+1}$$

$$\sqrt{5}$$

$$\boxed{(x+2)^2 + (y-1)^2 = 5}$$

$$3. \quad f(x) = 2x + 5$$

$$y = 2x + 5$$

$$x = 2y + 5$$

$$\frac{x-5}{2} = \frac{2y}{2}$$

$$\boxed{f^{-1}(x) = y = \frac{x-5}{2}}$$

$$4. \quad f(x) = \frac{x+4}{x-3}$$

$$y = \frac{x+4}{x-3}$$

$$x = \frac{y+4}{y-3}$$

$$(y-3)x = y+4$$

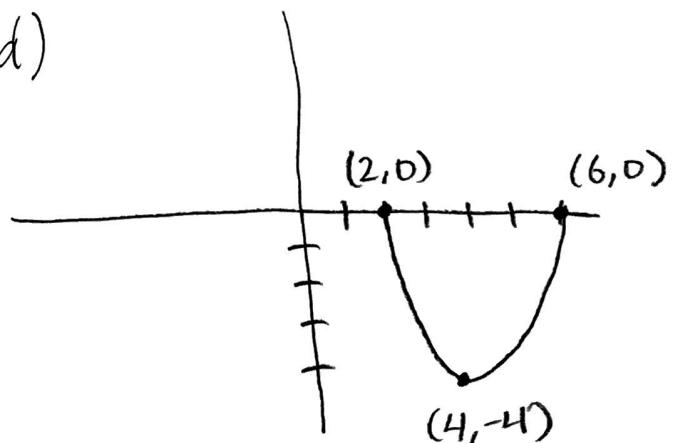
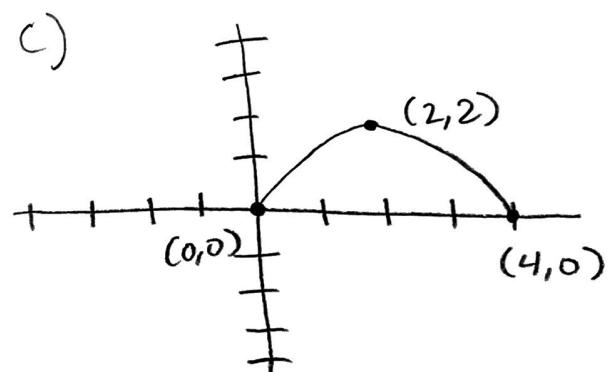
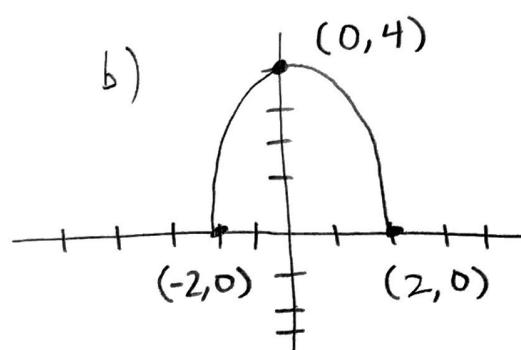
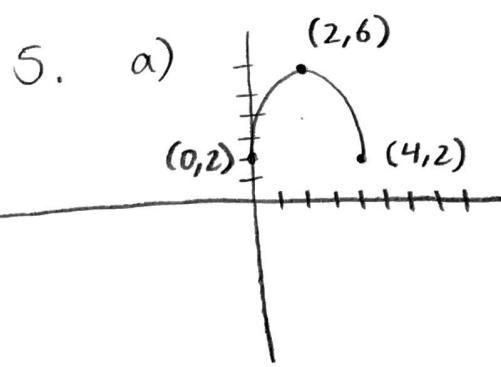
$$xy - 3x = y+4$$

$$-3x = -xy + y + 4$$

$$-4 - 3x = -xy + y$$

$$\frac{-4 - 3x}{(-x+1)} = \frac{y(-x+1)}{(-x+1)}$$

$$\boxed{f^{-1}(x) = y = \frac{-4 - 3x}{-x+1}}$$



$$6. \left(\frac{f}{g}\right)(x) = \frac{x^2 + 2x - 5}{\frac{2x+3}{x-2}}$$

$$(f \circ g)(x) = \left(\frac{2x+3}{x-2}\right)^2 + 2\left(\frac{2x+3}{x-2}\right) - 5$$

$$(g \circ f)(x) = \frac{2(x^2 + 2x - 5) + 3}{(x^2 + 2x - 5) - 2}$$

$$7. 125^\circ + 360^\circ = 485^\circ$$

$$485^\circ + 360^\circ = 845^\circ$$

$$125^\circ - 360^\circ = -235^\circ$$

$$-235^\circ - 360^\circ = -595^\circ$$

$$9. \alpha = 60^\circ = \frac{\pi}{3} \text{ rad}$$

$$s = \theta r$$

$$= \left(\frac{\pi}{3}\right)(20)$$

$$= \boxed{\frac{20\pi}{3} \text{ ft}}$$

$$8. 64^\circ 48' 32''$$

$$\begin{array}{r} + 39^\circ 38' 28'' \\ \hline 103^\circ 86' 60'' \end{array}$$

$$\begin{array}{r} + 1' - 60'' \\ \hline 87' 0'' \end{array}$$

$$\begin{array}{r} + 1^\circ - 60' \\ \hline 104^\circ 27' \end{array}$$

$$10. \sin(125^\circ) = 0.8192$$

$$\tan(4.7) = 80.7128$$

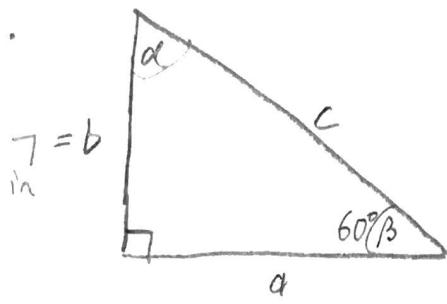
$$\cos\left(\frac{11\pi}{7}\right) = 0.2225$$

$$\sec(-68^\circ) = 2.6695$$

$$\frac{27 \text{ min}}{60 \text{ min}} \left| \frac{1 \text{ degree}}{60 \text{ min}} \right. = \frac{27}{60} \text{ degree} = 0.45 \text{ degree}$$

$$104^\circ + 0.4^\circ = \boxed{104.45^\circ}$$

11.



$$\alpha = 90^\circ - 60^\circ = 30^\circ$$

$$\text{angle } \alpha = 30^\circ$$

$$\sin 60^\circ = \frac{7}{c}$$

$$\frac{\sqrt{3}}{2} = \frac{7}{c}$$

$$\frac{c\sqrt{3}}{\sqrt{3}} = \frac{14}{\sqrt{3}}$$

$$\boxed{\text{side } c = \frac{14}{\sqrt{3}} \text{ in}}$$

$$\tan 60^\circ = \frac{7}{a}$$

$$\sqrt{3} = \frac{7}{a}$$

$$\frac{a\sqrt{3}}{\sqrt{3}} = \frac{7}{\sqrt{3}}$$

$$\boxed{\text{side } a = \frac{7}{\sqrt{3}} \text{ in}}$$

12. a)  $285^\circ \times \frac{\frac{57}{\pi}}{360^\circ} = \boxed{\frac{19\pi}{12}}$

$$160^\circ \times \frac{\frac{57}{\pi}}{360^\circ} = \boxed{\frac{8\pi}{9}}$$

b)  $\frac{7\pi}{81} \times \frac{\frac{36}{180^\circ}}{\pi} = \boxed{252^\circ}$

$$\frac{11\pi}{7} \times \frac{180^\circ}{\pi} = \boxed{\left(\frac{1980}{7}\right)^\circ \approx 282.9^\circ}$$

13. a)  $(+, +) \Rightarrow QII$

b)  $(-, -) \Rightarrow QIII$

14. a)  $\sin x = \frac{\sqrt{3}}{2}$

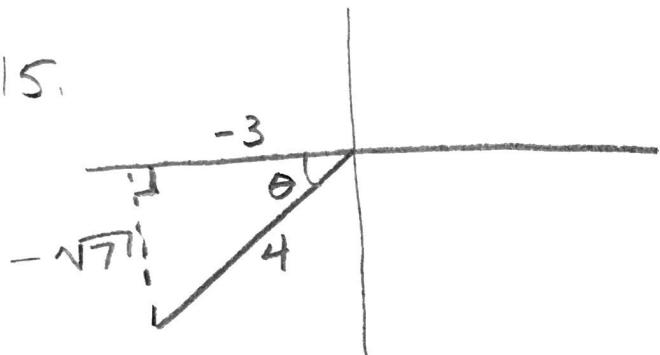
$$\boxed{x = 60^\circ}$$

b)  $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$\boxed{x = 30^\circ}$$

15.



$$(-3)^2 + b^2 = 4^2$$

$$9 + b^2 = 16$$

$$\sqrt{b^2} = \sqrt{7}$$

$$b = \sqrt{7}$$

$$\sin \theta = -\frac{\sqrt{7}}{4}$$

$$\csc \theta = -\frac{4}{\sqrt{7}}$$

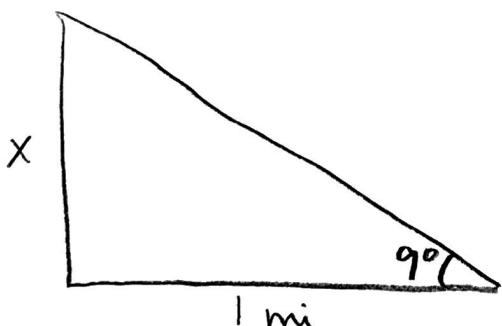
$$\cos \theta = -\frac{3}{4}$$

$$\sec \theta = -\frac{4}{3}$$

$$\tan \theta = \frac{\sqrt{7}}{3}$$

$$\cot \theta = \frac{3}{\sqrt{7}}$$

16.



$$\frac{0.1584 \text{ mi}}{1 \text{ mi}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = \boxed{836.352 \text{ ft}}$$

$$\tan 9^\circ = \frac{x}{1}$$

$$x = 0.1584 \text{ mi}$$

$$17. \quad v = r\omega$$

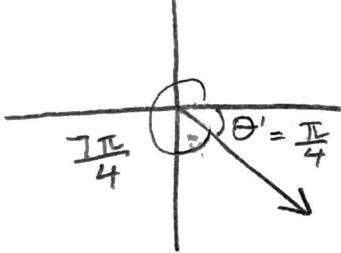
$$r = 0.5 \text{ in}$$

$$\frac{45,000 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{90000\pi \text{ rad}}{1 \text{ min}}$$

$$v = (90000\pi \text{ rad/min})(0.5 \text{ in/rad}) = 45000\pi \frac{\text{in}}{\text{min}}$$

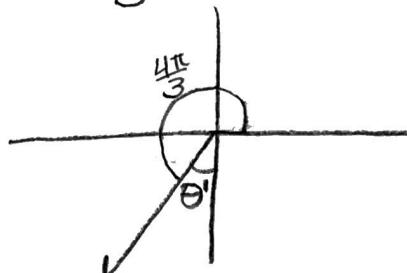
$$\frac{45000\pi \text{ in}}{\text{min}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 133.8747 \frac{\text{mi}}{\text{hr}}$$

$$18. \quad \frac{7\pi}{4} = 315^\circ$$



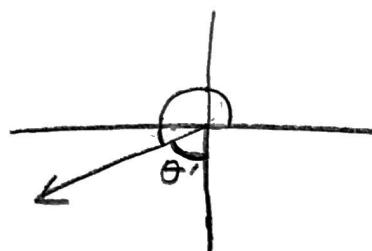
$$\theta' = \frac{8\pi}{4} - \frac{7\pi}{4} = \frac{\pi}{4}$$

$$\frac{4\pi}{3} = 240^\circ$$



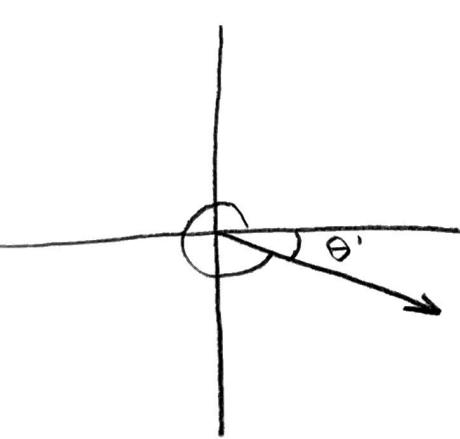
$$\theta' = \frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$$

$$210^\circ$$



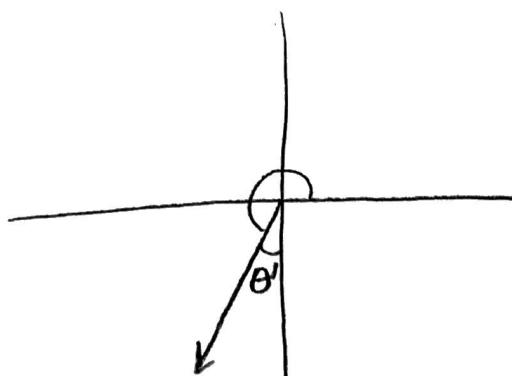
$$\theta' = 210^\circ - 180^\circ = 30^\circ$$

$$330^\circ$$



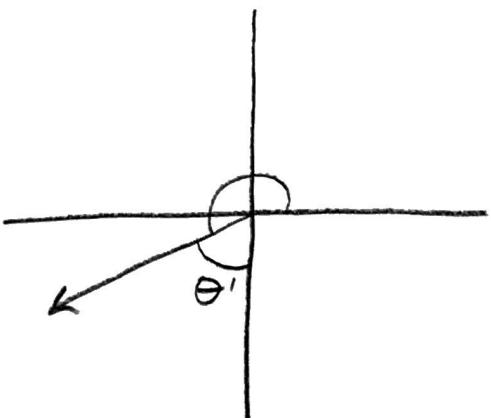
$$\theta' = 360^\circ - 330^\circ = 30^\circ$$

$$-120^\circ = 240^\circ$$



$$\theta' = 240^\circ - 180^\circ = 60^\circ$$

$$\frac{7\pi}{6} = 210^\circ$$



$$\theta' = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$$

$$18. \text{ a) } \sin\left(\frac{7\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

$$\text{b) } \cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = \boxed{-\frac{1}{2}}$$

$$\text{c) } \tan(210^\circ) = \tan(30^\circ) = \boxed{\frac{1}{\sqrt{3}}}$$

$$\text{d) } \cot(330^\circ) = -\cot(30^\circ) = \boxed{-\sqrt{3}}$$

$$\text{e) } \csc(-120^\circ) = -\csc(60^\circ) = \boxed{-\frac{2}{\sqrt{3}}}$$

$$\text{f) } \sec\left(\frac{7\pi}{6}\right) = -\sec\left(\frac{\pi}{6}\right) = \boxed{-2\sqrt{3}}$$

Name: \_\_\_\_\_

Total Received: \_\_\_\_\_

Show all work for full credit. NO CALCULATOR

1. State the amplitude, period, midline, and phase shift for each equation, and graph one cycle. List starting and ending points on  $x$ -axis. (6 + 6 = 12 Pt)

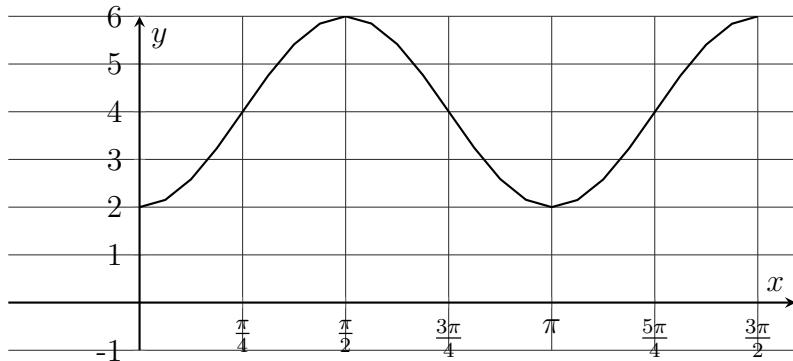
$$(a) y = -2 \sin[2(x - \frac{\pi}{4})] - 3 \quad (b) y = 3 \cos(2x) + 4$$

2. Sketch one period of the graph for each of the following functions. (12 Pts)

$$(a) f(x) = \tan(x + \frac{\pi}{4}) \quad (b) g(x) = \cot(\frac{1}{2}x) \quad (c) h(x) = 2 \csc(x + \frac{\pi}{4})$$

3. Find  $A, B, D$  and then write the equation for the graph in the form of

$$(i) y = A \sin[B(x - C)] + D \text{ and } (ii) y = A \cos[B(x - C)] + D \text{ (8 Pts)}$$



4. Write reciprocal identities, quotient identities, Pythagorean identities, and sum angle identities for sine and cosine. (10 Pts)

5. Verify the identities. (12 Pts)

$$(a) \frac{\sec x}{\tan x} - \frac{\tan x}{\sec x} = \cot x \cos x \quad (b) \frac{\csc x}{\cos x} - \frac{\cos x}{\sin x} = \tan x \\ (c) \sec x - \sin x \tan x = \cos x \quad (d) \sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

6. Find the **exact** values of (i)  $\sin(\alpha + \beta)$ , (ii)  $\cos(\alpha + \beta)$ , and (iii)  $\tan(\alpha + \beta)$  given that  $\cos \alpha = -\frac{3}{5}$  and  $\sin \beta = -\frac{5}{7}$ , with both  $\alpha$  and  $\beta$  are in quadrant III. (10 Pts)

7. Find the **exact** value of the indicated expression. (12 Pts)

$$(a) \sin 75^\circ \quad (b) \cos 105^\circ \quad (c) \sin 28^\circ \cos 17^\circ + \cos 28^\circ \sin 17^\circ$$

8. Find the **exact** value of the indicated expression. (8 Pts)

$$(a) \tan 22.5^\circ \quad (b) \cos 67.5^\circ$$

9. Find the **exact** value of  $\sin(2\alpha)$ ,  $\cos(2\alpha)$  and  $\tan(2\alpha)$  given that

$$\sin \alpha = \frac{8}{17}, \text{ with } \alpha \text{ in quadrant II. (8 Pts)}$$

10. Find the **exact** value of  $\sin(\frac{\alpha}{2})$ ,  $\cos(\frac{\alpha}{2})$  and  $\tan(\frac{\alpha}{2})$  given that  $\cot \alpha = \frac{5}{4}$ ,  $180^\circ < \alpha < 270^\circ$ . (8 Pts)

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### - Formulas -

1.  $\sin(2\alpha) = 2 \sin \alpha \cos \alpha, \quad \tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$
2.  $\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$
3.  $\sin(\frac{\alpha}{2}) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}, \quad \cos(\frac{\alpha}{2}) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}, \quad \tan(\frac{\alpha}{2}) = \frac{1 - \cos \alpha}{\sin \alpha}$

$$1. \text{ (a)} \quad y = -2 \sin \left[ 2 \left( x - \frac{\pi}{4} \right) \right] - 3$$

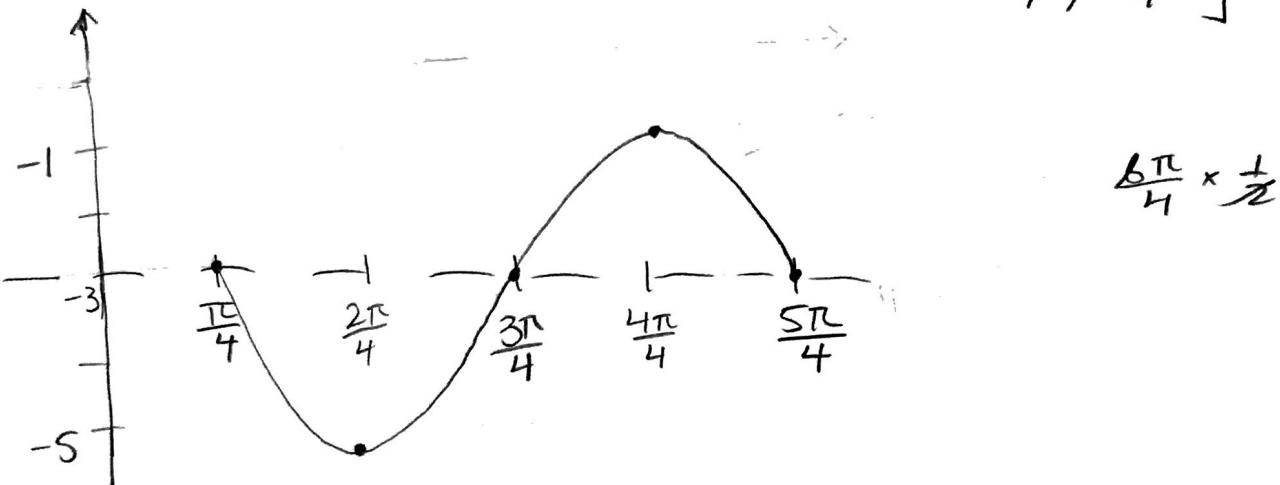
$$\text{amplitude} = |-2| = 2$$

$$\text{midline} = y = -3$$

$$\text{period} = \frac{2\pi}{2} = \pi \quad ; \text{ one cycle: } [0, \pi]$$

$$\text{phase shift} = +\frac{\pi}{4}$$

$$\begin{array}{r} +\frac{\pi}{4} \\ +\frac{\pi}{4} \\ \hline \frac{\pi}{4}, \frac{5\pi}{4} \end{array}$$



$$\text{(b)} \quad y = 3 \cos(2x) + 4$$

$$\text{amplitude} = |3| = 3$$

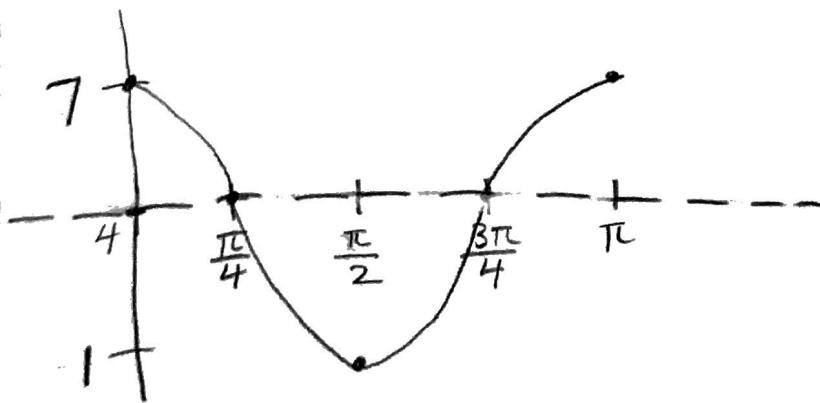
$$\text{midline} = y = 4$$

$$\text{Period} = \frac{2\pi}{2} = \pi \quad ; \text{ one cycle: } [0, \pi]$$

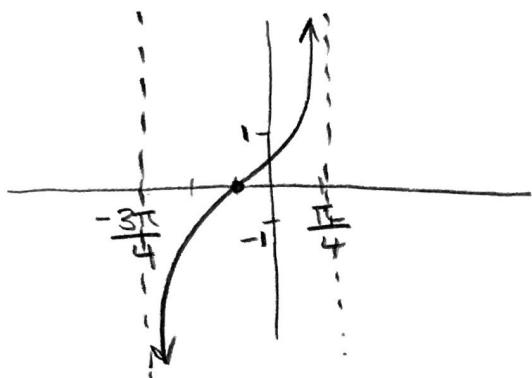
no phase shift

$$\frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$$

$$\frac{\pi}{2} + \frac{2\pi}{2} = \frac{3\pi}{2} \times \frac{1}{2}$$



$$2. (a) f(x) = \tan\left(x + \frac{\pi}{4}\right)$$



$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$-\frac{\pi}{4} \quad -\frac{\pi}{4}$$

$$-\frac{2\pi}{4} - \frac{\pi}{4} = -\frac{3\pi}{4}$$

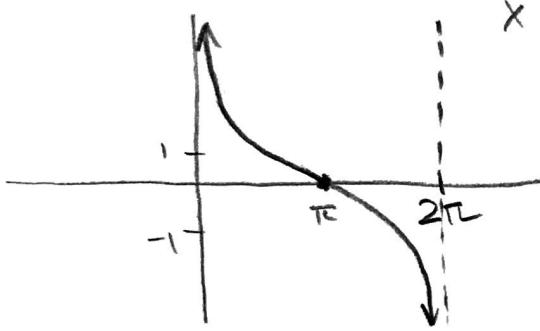
$$\frac{2\pi}{4} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\frac{\pi}{4} - \frac{3\pi}{4} = -\frac{2\pi}{4} \times \frac{1}{2} = -\frac{\pi}{4}$$

$$(b) g(x) = \cot\left(\frac{1}{2}x\right)$$

$$\cancel{\frac{1}{2}}x = 0$$

$$x = 0$$



$$\cancel{\frac{1}{2}}x = \frac{\pi}{2}$$

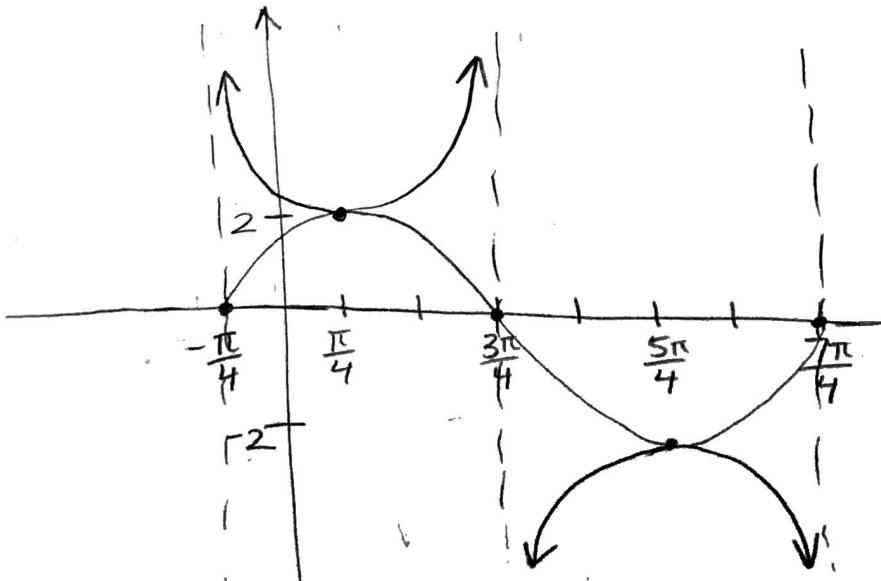
$$x = 2\pi$$

Period:  $\frac{\pi}{\frac{1}{2}} = \pi$   
 $\pi \cdot 2 = 2\pi$

$$2\pi + 0 = 2\pi \times \frac{1}{\frac{1}{2}}$$

$$(0, 2\pi)$$

$$(c) h(x) = 2\csc\left(x + \frac{\pi}{4}\right)$$



$$2\pi = \frac{8\pi}{4}$$

$$[0, 2\pi]$$

$$-\frac{\pi}{4} \quad -\frac{\pi}{4}$$

$$\left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$$

$$\frac{7\pi}{4} - \frac{\pi}{4} = \frac{6\pi}{4} \times \frac{1}{2} = \frac{3\pi}{4}$$

$$\frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4} \times \frac{1}{2} = \frac{\pi}{4}$$

$$\frac{\pi}{4} + \frac{3\pi}{4} = \frac{4\pi}{4} \times \frac{1}{2} = \frac{2\pi}{4}$$

$$3. \quad D = 4$$

$$A = 12$$

$$\text{Period} = \pi = \frac{2\pi}{B}$$

$$B\pi = 2\pi$$

$$B = 2$$

$$y = A \sin[B(x - c)] + D$$

$$y = 2 \sin\left[2\left(x - \frac{\pi}{4}\right)\right] + 4 \Rightarrow c = \frac{\pi}{4}$$

$$y = A \cos[B(x - c)] + D$$

$$y = -2 \cos(2x) + 4 \Rightarrow c = 0$$

#### 4. reciprocal identities

$$\frac{1}{\sin x} = \csc x \quad \frac{1}{\csc x} = \sin x$$

$$\frac{1}{\cos x} = \sec x \quad \frac{1}{\sec x} = \cos x$$

$$\frac{1}{\tan x} = \cot x \quad \frac{1}{\cot x} = \tan x$$

#### quotient identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

#### Sum Angle Identities

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

#### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\tan^2 x + 1 = \sec^2 x \rightarrow 1 = \sec^2 x - \tan^2 x$$

$$\rightarrow \tan^2 x = \sec^2 x - 1$$

$$1 + \cot^2 x = \csc^2 x \rightarrow \cot^2 x = \csc^2 x - 1$$

$$1 = \csc^2 x - \cot^2 x$$

5. a)  $\frac{\sec x}{\tan x} - \frac{\tan x}{\sec x} = \cot x \cos x$

L.S.:

$$\left( \frac{\sec x}{\sec x} \right) \frac{\sec x}{\tan x} - \frac{\tan x}{\sec x} \left( \frac{\tan x}{\tan x} \right)$$

$$= \frac{\sec^2 x - \tan^2 x}{\tan x \sec x}$$

$$= \frac{1}{\tan x \sec x} = \cot x \cos x = R.S.$$

c)  $\sec x - \sin x \tan x = \cos x$

L.S.  $\sec x - \sin x \tan x$

$$\frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \frac{\cancel{\cos x} \cos x}{\cancel{\cos x}} = \cos x = R.S.$$

b)

$$\frac{\csc x}{\cos x} - \frac{\cos x}{\sin x} = \tan x$$

L.S.  $\left( \frac{\sin x}{\sin x} \right) \frac{\csc x}{\cos x} - \frac{\cos x}{\sin x} \left( \frac{\cos x}{\cos x} \right)$

$$\frac{\sin x \csc x - \cos^2 x}{\cos x \sin x} = \frac{1 - \cos^2 x}{\cos x \sin x} = \frac{\sin^2 x}{\cos x \sin x} = \frac{\cancel{\sin x} \sin x}{\cancel{\cos x} \sin x}$$

$$= \frac{\sin x}{\cos x} = \tan x = R.S.$$

$$5d. \sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

(IG)

$$L.S. \quad \sec x + \tan x$$

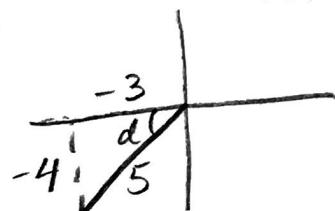
$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos x} \left( \frac{1 - \sin x}{1 - \sin x} \right)$$

$$= \frac{1 - \sin^2 x}{\cos x (1 - \sin x)} = \frac{\cos^2 x}{\cos x (1 - \sin x)} = \frac{\cancel{\cos x} \cos x}{\cancel{\cos x} (1 - \sin x)}$$

$$= \frac{\cos x}{1 - \sin x} = R.S.$$

$$6. \cos \alpha = -\frac{3}{5}, \quad \sin \beta = -\frac{5}{7}$$



$$(-3)^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$



$$(-5)^2 + b^2 = 7^2$$

$$25 + b^2 = 49$$

$$\sqrt{b^2} = \sqrt{24}$$

(continued on a  
separate page)

6. (continued)

(IG)

$$\begin{aligned}\sin(\alpha + \beta) &= \sin\alpha \cos\beta + \cos\alpha \sin\beta \\&= \left(+\frac{4}{5}\right)\left(+\frac{\sqrt{24}}{7}\right) + \left(+\frac{3}{5}\right)\left(+\frac{5}{7}\right) \\&= \frac{4\sqrt{24}}{35} + \frac{15}{35} = \boxed{\frac{4\sqrt{24} + 15}{35}}\end{aligned}$$

$$\begin{aligned}\cos(\alpha + \beta) &= \cos\alpha \cos\beta - \sin\alpha \sin\beta \\&= \left(+\frac{3}{5}\right)\left(+\frac{\sqrt{24}}{7}\right) - \left(-\frac{4}{5}\right)\left(-\frac{5}{7}\right) \\&= \frac{3\sqrt{24}}{35} - \frac{20}{35} = \frac{3\sqrt{24} - 20}{35}\end{aligned}$$

$$\begin{aligned}\tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\frac{4\sqrt{24} + 15}{35}}{\frac{3\sqrt{24} - 20}{35}} \\&= \frac{4\sqrt{24} + 15}{35} \times \frac{35}{3\sqrt{24} - 20} \\&= \boxed{\frac{4\sqrt{24} + 15}{3\sqrt{24} - 20}}\end{aligned}$$

$$7. \text{ (a)} \sin 75^\circ = \sin(30^\circ + 45^\circ)$$

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\sin(30^\circ + 45^\circ) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

Q II  
 $\cos = -$

$$\text{(b)} \cos 105^\circ = -\cos(60^\circ + 45^\circ)$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos(60^\circ + 45^\circ) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

negative

$$\text{(c)} \sin 28^\circ \cos 17^\circ + \cos 28^\circ \sin 17^\circ$$

$$\sin(28^\circ + 17^\circ) = \sin(45^\circ) = \boxed{\frac{\sqrt{2}}{2}}$$

$$8. (a) \tan 22.5^\circ = \tan\left(\frac{45^\circ}{2}\right)$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos\alpha}{\sin\alpha}$$

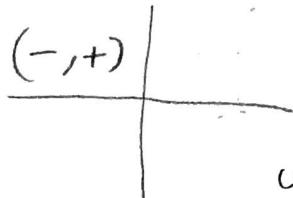
$$\tan\left(\frac{45^\circ}{2}\right) = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{1}{2} - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \boxed{\frac{2 - \sqrt{2}}{\sqrt{2}}}$$

$\nearrow QII$   
 $\theta' = 180^\circ - 135^\circ = 45^\circ$

QI  
 $\cos = (+)$

$$(b) \cos 67.5^\circ = \cos \frac{135^\circ}{2}$$



$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos\alpha}{2}}$$

$$\cos 135^\circ = -\cos 45^\circ$$

$$\cos\left(\frac{135^\circ}{2}\right) = \sqrt{\frac{1 + \left(-\frac{\sqrt{2}}{2}\right)}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{2} \times \frac{1}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \boxed{\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

$$9. \sin \alpha = \frac{8}{17},$$

$$90^\circ < \alpha < 180^\circ$$

$\times 2$

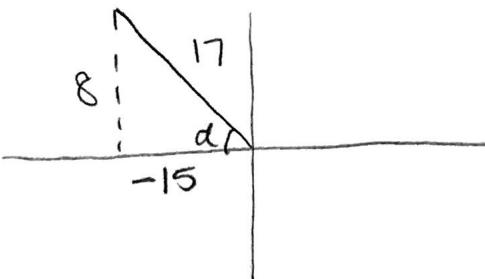
$\times 2$

$\times 2$

$$180^\circ < 2\alpha < 360^\circ$$

$\hookrightarrow Q III \text{ or } Q IV$

$$\sin = (-)$$



$$8^2 + b^2 = 17^2$$

$$\cancel{64} + b^2 = \cancel{289}$$

$$\sqrt{b^2} = \sqrt{225}$$

$$b = 15$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$= 2 \left( \frac{8}{17} \right) \left( -\frac{15}{17} \right)$$

$$= \boxed{-\frac{240}{289}}$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$= \left( -\frac{15}{17} \right)^2 - \left( \frac{8}{17} \right)^2$$

QIV

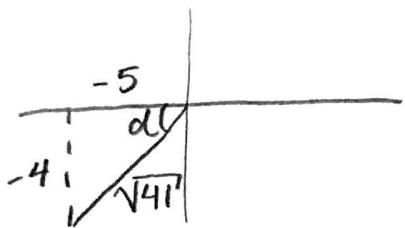
$$\tan(2\alpha) = \frac{\sin(2\alpha)}{\cos(2\alpha)} = -\frac{\frac{240}{289}}{\frac{161}{289}}$$

$$\frac{240}{289} - \frac{161}{289} = \boxed{\frac{161}{289}}$$

$$= -\frac{240}{289} \times \frac{289}{161} = \boxed{-\frac{240}{161}}$$

$$10. \cot \alpha = \frac{5}{4} ; \quad \frac{180^\circ}{2} < \frac{\alpha}{2} < \frac{270^\circ}{2}$$

(IG)   
 (-, +)



$$90^\circ < \frac{\alpha}{2} < 135^\circ \Rightarrow QII$$

$$\sin = (+) \\ \cos = (-)$$

$$(-5)^2 + (-4)^2 = c^2$$

$$25 + 16 = c^2$$

$$\sqrt{41} = \sqrt{c^2}$$

$$\sin\left(\frac{\alpha}{2}\right) = + \sqrt{\frac{1 - \cos\alpha}{2}}$$

$$= \sqrt{\frac{1 - \left(-\frac{5}{\sqrt{41}}\right)}{2}} = \sqrt{\frac{\frac{\sqrt{41}}{\sqrt{41}} + \frac{5}{\sqrt{41}}}{2}} = \sqrt{\frac{\frac{\sqrt{41} + 5}{\sqrt{41}} \times \frac{1}{2}}{\frac{\sqrt{41} + 5}{2\sqrt{41}}}}$$

$$\cos\left(\frac{\alpha}{2}\right) = - \sqrt{\frac{1 + \cos\alpha}{2}}$$

$$= - \sqrt{\frac{1 + \left(\frac{-5}{\sqrt{41}}\right)}{2}} = - \sqrt{\frac{\frac{\sqrt{41}}{\sqrt{41}} - \frac{5}{\sqrt{41}}}{2}} = - \sqrt{\frac{\frac{\sqrt{41} - 5}{\sqrt{41}} \times \frac{1}{2}}{\frac{\sqrt{41} - 5}{2\sqrt{41}}}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)} = \frac{\sqrt{\frac{\sqrt{41} + 5}{2\sqrt{41}}}}{-\sqrt{\frac{\sqrt{41} - 5}{2\sqrt{41}}}}$$

$$= \frac{\sqrt{\sqrt{41} + 5}}{\sqrt{2\sqrt{41}}} \times -\frac{\sqrt{2\sqrt{41}}}{\sqrt{\sqrt{41} - 5}} = -\sqrt{\frac{\sqrt{41} + 5}{\sqrt{41} - 5}}$$

**Final Exam MTH 122 Fall 2022 Total Pts:100 12/5/2022**

Name: \_\_\_\_\_ Total Received:

Show all work for full credit. Write all your solutions on the blank papers.

1. Find the **exact** value of each expression without using calculator. (12 Pts)
  - (a)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
  - (b)  $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
  - (c)  $\cos(\cot^{-1}\left(\frac{4}{3}\right))$
  - (d)  $\sin(\cos^{-1}\left(\frac{-5}{12}\right))$
2. Find the **exact** solutions over the indicated intervals. (20 Pts)
  - (a)  $2 \cos x - \sqrt{3} = 0$ , all solutions
  - (b)  $\tan x - \sqrt{3} = 0$ , all solutions
  - (c)  $2\sin(2x) - \sqrt{3} = 0$ ,  $0^\circ \leq x < 360^\circ$
  - (d)  $2 \cos^2 x + 3 \cos x + 1 = 0$ , all solutions
  - (e)  $4 \sin^2 x - 3 = 0$ , all solutions
3. **Write** the law of sines and the law of cosines. (5 Pts)
4. Use the law of sines or the law of cosines to solve each triangle. (20 Pts)
  - (a)  $A = 50^\circ$ ,  $C = 75^\circ$ ,  $a = 10$
  - (b)  $A = 70^\circ$ ,  $b = 9$ ,  $c = 11$
  - (c)  $a = 3$ ,  $b = 6$ ,  $c = 9$
  - (d)  $B = 60^\circ$ ,  $a = 4$ ,  $b = 6$
5. Find the area of each triangle. (8 Pts)
  - (a)  $a = 5 \text{ in}$ ,  $b = 7 \text{ in}$ ,  $c = 4 \text{ in}$ ,
  - (b)  $B = 44^\circ$ ,  $a = 5$ ,  $b = 8$
6. Consider the following points in the plane: A(2,3), B(-2,1), C(1,2), D(-1,-1)
  - (4 Pts) (a) Find the vectors  $\vec{u} = \overrightarrow{AB}$ ,  $\vec{v} = \overrightarrow{BC}$ ,  $\vec{w} = \overrightarrow{CD}$ .
  - (4 Pts) (b) Find  $\vec{u} + \vec{v}$ ,  $3\vec{u} - 2\vec{v}$ ,  $2\vec{u} + \vec{v} + 3\vec{w}$ .
  - (4 Pts) (c) Find  $|\vec{u}|$ ,  $|\vec{v}|$ ,  $|\vec{w}|$ .
  - (4 Pts) (d) Find the dot products  $\vec{u} \cdot \vec{v}$ ,  $\vec{v} \cdot \vec{w}$ .
  - (4 Pts) (e) Find the angle between the vectors  $\vec{u}$  and  $\vec{v}$ .
7. Consider two complex numbers  $z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ ,  $z_2 = -1 + 1i$ . (8 Pts)
  - (a) Change them to polar form  $r(\cos \theta + i \sin \theta)$ .
  - (b) Use the polar forms of  $z_1$  and  $z_2$  to find  $z_1 z_2$  and  $\frac{z_1}{z_2}$ .
8. Consider the complex number  $z = 1 + 1i$ . (7 Pts)
  - (a) Change it to polar form  $r(\cos \theta + i \sin \theta)$ .
  - (b) Use the polar form of  $z$  to find  $(z)^8 = (1 + 1i)^8$ .

|                      | $0^\circ$            | $30^\circ$           | $45^\circ$           | $60^\circ$           | $90^\circ$ |
|----------------------|----------------------|----------------------|----------------------|----------------------|------------|
| $\sqrt{\frac{0}{4}}$ | $\sqrt{\frac{1}{4}}$ | $\sqrt{\frac{2}{4}}$ | $\sqrt{\frac{3}{4}}$ | $\sqrt{\frac{4}{4}}$ |            |
| sin                  | 0                    | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1          |
| cos                  | 1                    | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | 0          |
| tan                  | 0                    | $\frac{1}{\sqrt{3}}$ | 1                    | $\sqrt{3}$           | undefined  |

1. a)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{30^\circ}$

~~11/22~~

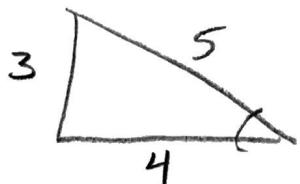
b)  $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \boxed{330^\circ}$        $360^\circ - 30^\circ = 330^\circ$

~~11/22~~

c)  $\cos(\cot^{-1}\left(\frac{4}{3}\right)) = \cos\theta = \boxed{\frac{4}{5}}$

$\cot^{-1}\left(\frac{4}{3}\right) = \theta$

$\hookrightarrow \cot\theta = \frac{4}{3}$



$4^2 + 3^2 = c^2$

$16 + 9 = c^2$

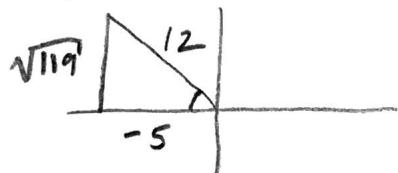
$\sqrt{25} = \sqrt{c^2}$

$c = 5$

$$1. (d) \sin(\cos^{-1}\left(-\frac{5}{12}\right)) = \sin\theta = \boxed{\frac{\sqrt{119}}{12}}$$

$$\cos^{-1}\left(-\frac{5}{12}\right) = \theta$$

$$\Leftrightarrow \cos\theta = -\frac{5}{12}$$



$$(-5)^2 + b^2 = 12^2$$

$$25 + b^2 = 144$$

$$\sqrt{b^2} = \sqrt{119}$$

$$2. \text{ a) } 2\cos x - \sqrt{3} = 0, \text{ all solutions}$$

$$\frac{2\cos x}{2} = \frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\begin{array}{c} | \\ + \\ \hline | \\ + \end{array}$$

$30^\circ$

$$360^\circ - 30^\circ = 330^\circ$$

$$\boxed{x = 30^\circ + 360^\circ K}$$

$$\boxed{x = 330^\circ + 360^\circ K}$$

$$\text{b) } \tan x - \sqrt{3} = 0, \text{ all solutions}$$

$$\tan x = \sqrt{3}$$

$$\begin{array}{c} | \\ + \\ \hline | \\ + \end{array}$$

$$60^\circ$$

$$\boxed{x = 60^\circ + 360^\circ K}$$

$$\boxed{x = 240^\circ + 360^\circ K}$$

$$180^\circ + 60^\circ = 240^\circ$$

$$\underline{\underline{60^\circ + 180^\circ K}}$$

$$2. \text{ c) } 2\sin(2x) - \sqrt{3} = 0, \quad 0^\circ \leq x < 360^\circ$$

$$\frac{2\sin(2x)}{2} = \frac{\sqrt{3}}{2}$$

$$\sin(2x) = \frac{\sqrt{3}}{2}$$

$$\begin{array}{c} + | + \\ \hline 60^\circ \\ 180^\circ - 60^\circ = 120^\circ \end{array}$$

$$\frac{2x}{2} = \frac{60^\circ + 360^\circ K}{2}$$

$$x = 30^\circ + 180^\circ K$$

$$\frac{2x}{2} = \frac{120^\circ + 360^\circ K}{2}$$

$$x = 60^\circ + 180^\circ K$$

$$\boxed{K=0 : 30^\circ, 60^\circ}$$

$$K=1 : 210^\circ, 240^\circ$$

$$d) \quad 2\cos^2x + 3\cos x + 1 = 0, \text{ all solutions}$$

$$2\cos^2x + 2\cos x + \cos x + 1 = 0$$

$$2\cos x (\cos x + 1) + 1(\cos x + 1) = 0$$

$$(2\cos x + 1)(\cos x + 1) = 0$$

$$2\cos x + 1 = 0$$

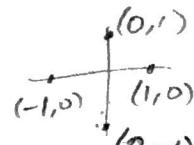
$$\frac{2\cos x}{2} = -\frac{1}{2}$$

$$\cos x = -\frac{1}{2}$$

$$\cos x + 1 = 0$$

$$\cos x = -1$$

$$\boxed{x = 180^\circ + 360^\circ K}$$



$$\boxed{x = 120^\circ + 360^\circ K}$$

$$x = 240^\circ + 360^\circ K$$

$$180^\circ - 60^\circ = 120^\circ$$

$$180^\circ + 60^\circ = 240^\circ$$

2. e)  $4 \sin^2 x - 3 = 0$ , all solutions

$$\frac{4 \sin^2 x}{4} = \frac{3}{4}$$

$$\sqrt{\sin^2 x} = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

|   |   |
|---|---|
| + | + |
| - | - |

$$x = 60^\circ + 360^\circ K$$

$$x = 300^\circ + 360^\circ K$$

$$x = 120^\circ + 360^\circ K$$

$$x = 240^\circ + 360^\circ K$$

3. Law of sines =  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of cosines =  $c^2 = a^2 + b^2 - 2ab \cos C$   
 $b^2 = a^2 + c^2 - 2ac \cos B$   
 $a^2 = b^2 + c^2 - 2bc \cos A$

4. a)  $A = 50^\circ$ ,  $C = 75^\circ$ ,  $a = 10$

$$B = 180^\circ - (50^\circ + 75^\circ) = \boxed{55^\circ}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{10}{\sin 50^\circ} = \frac{b}{\sin 55^\circ}$$

$$b = \frac{10 \sin 55^\circ}{\sin 50^\circ} = \boxed{10.69}$$

4a. (continued)

$$\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{10}{\sin 50^\circ} = \frac{c}{\sin 75^\circ}$$

$$c = \frac{10 \sin 75^\circ}{\sin 50^\circ} = \boxed{12.61}$$

b)  $A = 70^\circ$ ,  $b = 9$ ,  $c = 11$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 9^2 + 11^2 - 2(9)(11) \cos 70^\circ$$

$$\sqrt{a^2} = \sqrt{134.28}$$

$$\boxed{a = 11.59}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{11.59}{\sin 70^\circ} = \frac{9}{\sin B}$$

$$11.59 \sin B = 9 \sin 70^\circ$$

$$\sin B = \frac{9 \sin 70^\circ}{11.59}$$

$$B = \sin^{-1} \left( \frac{9 \sin 70^\circ}{11.59} \right) = \boxed{46.86^\circ}$$

$$C = 180^\circ - (70^\circ + 46.86^\circ) = \boxed{63.14^\circ}$$

$$4. \text{ c) } a=3, b=6, c=9$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$9^2 = 3^2 + 6^2 - 2(3)(6) \cos C$$

$$81 = 45 - 36 \cos C$$

$$\frac{36}{-36} = \frac{-36 \cos C}{-36}$$

$$\cos C = -1$$

$$C = \cos^{-1}(-1) = 180^\circ$$

→ Triangle does not exist.

$$\text{d) } B = 60^\circ, a = 4, b = 6$$

$$\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{6}{\sin 60^\circ} = \frac{4}{\sin A}$$

$$\frac{6 \sin A}{6} = \frac{4 \sin 60^\circ}{6}$$

$$A = \sin^{-1} \left( \frac{4 \sin 60^\circ}{6} \right) = \boxed{35.26^\circ}$$

$$C = 180^\circ - (60^\circ + 35.26^\circ) = \boxed{84.74^\circ}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} = \frac{6}{\sin 60^\circ} = \frac{c}{\sin C}$$

$$c = \frac{6 \sin 84.74^\circ}{\sin 60^\circ} = \boxed{6.899}$$

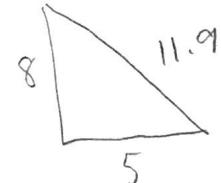
$$5. \text{ a)} \quad a = 5, \quad b = 7, \quad c = 4$$

$$s = \frac{5+7+4}{2} = 8$$

$$\begin{aligned}\text{Area} &= \sqrt{8(8-5)(8-7)(8-4)} \\ &= \boxed{9.798 \text{ in}^2}\end{aligned}$$

$$\text{b)} \quad B = 44^\circ, \quad a = 5, \quad b = 8$$

$$\frac{b}{\sin B} = \frac{a}{\sin A} = \frac{8}{\sin 44^\circ} = \frac{5}{\sin A}$$



$$\frac{8 \sin A}{8} = \frac{5 \sin 44^\circ}{8}$$

$$A = \sin^{-1} \left( \frac{5 \sin 44^\circ}{8} \right) = 25.73^\circ$$

$$C = 180^\circ - (44^\circ + 25.73^\circ) = 110.27^\circ$$

$$180^\circ - (10.27^\circ) = 69.73^\circ$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} = \frac{8}{\sin 44^\circ} = \frac{c}{\sin 110.27^\circ}$$

$$c = \frac{8 \sin 110.27^\circ}{\sin 44^\circ} = 10.8$$

$$s = \frac{10.8 + 5 + 8}{2} = 11.9$$

$$\text{Area} = \sqrt{11.9(11.9-5)(11.9-8)(11.9-10.8)} = \boxed{18.768 \text{ units}^2}$$

QII  
will result in  
(-) area

$$\begin{aligned}\text{Area} &= \frac{1}{2}(a)(b)\cos C \\ &= \frac{1}{2}(5)(8)\cos(69.73^\circ)\end{aligned}$$

$$= \boxed{6.929 \text{ units}^2}$$

$$\begin{aligned}\text{Area} &= \frac{1}{2}(a)(b)\sin C \\ &\Leftrightarrow \boxed{18.761 \text{ units}^2}\end{aligned}$$

I might have misremembered formula.

$$6. \text{ a) } \vec{u} = \overrightarrow{AB} = \langle (-2)-2, 1-3 \rangle = \boxed{\langle -4, -2 \rangle}$$

$$\vec{v} = \overrightarrow{BC} = \langle 1-(-2), 2-1 \rangle = \boxed{\langle 3, 1 \rangle}$$

$$\vec{w} = \overrightarrow{CD} = \langle (-1)-1, (-1)-2 \rangle = \boxed{\langle -2, -3 \rangle}$$

$$\text{b) } \vec{u} + \vec{v} = \langle -4+3, -2+1 \rangle = \boxed{\langle -1, -1 \rangle}$$

$$3\vec{u} - 2\vec{v} = \langle -12, -6 \rangle - \langle 6, 2 \rangle$$

$$= \langle -12-6, -6-2 \rangle = \boxed{\langle -18, -8 \rangle}$$

$$2\vec{u} + \vec{v} + 3\vec{w} = \langle -8, -4 \rangle + \langle 3, 1 \rangle + \langle -6, -9 \rangle$$

$$= \langle -8+3, -4+1 \rangle = \langle -5, -3 \rangle$$

$$= \langle -5+(-6), -3+(-9) \rangle = \boxed{\langle -11, -12 \rangle}$$

$$\text{c) } |\vec{u}| = \sqrt{(-4)^2 + (-2)^2} = \sqrt{16+4} = \boxed{\sqrt{20}}$$

$$|\vec{v}| = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \boxed{\sqrt{10}}$$

$$|\vec{w}| = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \boxed{\sqrt{13}}$$

$$6. d) \quad \vec{u} \cdot \vec{v} = (-4)(3) + (-2)(1)$$

$$-12 + (-2) = \boxed{-14}$$

$$\vec{v} \cdot \vec{w} = (3)(-2) + (1)(-3)$$

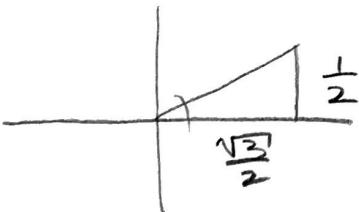
$$-6 + (-3) = \boxed{-9}$$

$$e) \quad \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

$$-14 = \sqrt{20} \cdot \sqrt{10} \cos \theta$$

$$\cos \theta = \frac{-14}{\sqrt{20} \cdot \sqrt{10}}$$

$$\theta = \cos^{-1} \left( \frac{-14}{\sqrt{20} \cdot \sqrt{10}} \right) = \boxed{171.87^\circ}$$

$$7. \quad z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$


$$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = r^2$$

$$\frac{3}{4} + \frac{1}{4} = r^2$$

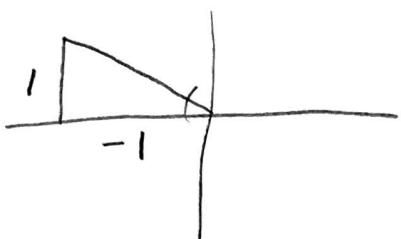
$$r = 1$$

$$\tan \theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) = 30^\circ$$

$$z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i = \boxed{(\cos 30^\circ + i \sin 30^\circ)}$$

$$7. \quad z_2 = -1 + i$$



QII

$$(-1)^2 + (1)^2 = r^2$$

$$1 + 1 = r^2$$

$$\sqrt{2} = \sqrt{r^2}$$

$$r = \sqrt{2}$$

$$\tan \theta = \frac{1}{-1} = -1$$

$$\theta = \tan^{-1}(-1) = -45^\circ \quad \text{wrong quadrant}$$

$$\theta = 180^\circ - 45^\circ = 135^\circ$$

$$z_2 = -1 + i = \boxed{\sqrt{2} (\cos 135^\circ + i \sin 135^\circ)}$$

$$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

$$= (1)(\sqrt{2}) [\cos(30^\circ + 135^\circ) + i \sin(30^\circ + 135^\circ)]$$

$$= \sqrt{2} [\cos(165^\circ) + i \sin(165^\circ)]$$

$$= \sqrt{2} [-0.966 + i(0.259)] \quad \text{QI}$$

$$= \boxed{-1.366 + 0.366i}$$

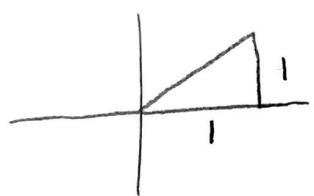
$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$$

$$= \frac{1}{\sqrt{2}} [\cos(30^\circ - 135^\circ) + i \sin(30^\circ - 135^\circ)] \quad \text{QIII}$$

$$= \frac{1}{\sqrt{2}} [\cos(-105^\circ) + i \sin(-105^\circ)]$$

$$= \frac{1}{\sqrt{2}} [-0.259 + i(-0.966)] = \boxed{-0.183 - 0.683i}$$

$$8. \quad z = 1 + i$$



$$(1)^2 + (1)^2 = r^2$$

$$1 + 1 = r^2$$

$$\sqrt{2} = \sqrt{r^2}$$

QI

$$r = \sqrt{2}$$

$$\tan \theta = \frac{1}{1} = 1$$

$$\theta = \tan^{-1}(1) = 45^\circ$$

$$z = 1+i = \sqrt{2} (\cos 45^\circ + i \sin 45^\circ)$$

$$z^n = r^n [\cos(n\theta) + i \sin(n\theta)]$$

$$z^8 = (\sqrt{2})^8 [\cos(8 \times 45^\circ) + i \sin(8 \times 45^\circ)]$$

$$= 16 [\cos(360^\circ) + i \sin(360^\circ)]$$

$$= 16 [1 + 0i]$$

$$= 16 + (16)(0)i = 16 + 0i = 16$$

