Exam 1 Math 132 Spring 2023 Total Pts:100 2/3/2023

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1. (**12** Pts) Solve the following equations for x.
   (a) $-2(3x+2)-4=2(x-1)+3x$ (b) $x^2+7x+12=0$
   (c) $2x^2-x-6=0$ (d) $4x^2-9=0$

2. (**10** Pts) Solve the following equations for x.
   (a) $x^4-10x^2+9=0$ (b) $x^3-2x^2-4x+8=0$ (c) $\sqrt{x+6}=x$

3. (**8** Pts)Perform the indicated operations and simplify.
   (a) $\frac{2x+1}{x+2}-\frac{x-4}{x^2+5x+6}$ (b) $\frac{x^2-5x-6}{x^2+5x+6}\div\frac{x^2-x-6}{x^2-4}$

4. (**4** Pts) Compute the average rate of change of $f(x)=2x^2-3x+5$ on the interval $[0,4]$.

5. (**6** Pts) Evaluate the given expressions and simplify.
   (a) $(3-3i)-(4-2i)$ (b) $(3-3i)(4-2i)$ (c) $\frac{3-3i}{4-2i}$

6. (**4** Pts) Find the domain for the following functions:
   (a) $f(x)=\frac{2x+3}{3x-2}$ (b) $g(x)=\sqrt{2x-5}$

7. (**8** Pts) (a) Find an equation for the line that is passing through (-2,3) and is parallel to the line $3x-4y=1$. (b) Also, find another equation for the line that is passing through (1,-2) and is perpendicular to the line $3x-4y=1$.

8. (**8** Pts) For the functions $f(x)=x^2+3x-4$ and $g(x)=\frac{2x+3}{x-2}$, find the following.
   (a) $(f+g)(3)$ (b) $(f.g)(x)$ (c) $(f\circ g)(x)$ (d) $(g\circ f)(x)$ (e) $(f\circ f)(2)$

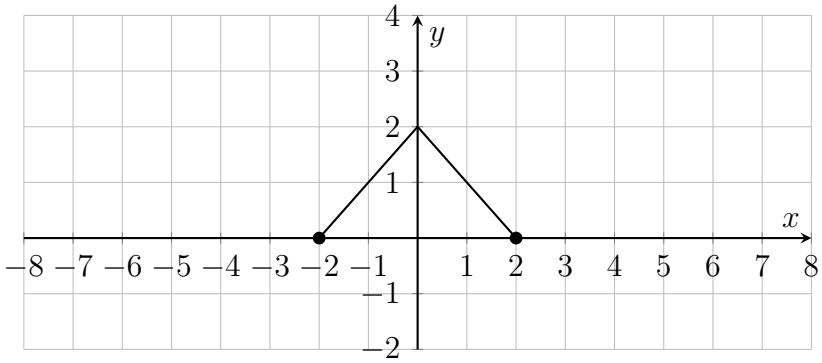
9. (**10** Pts) Solve the following.
   (a) $|x|=2$ (b) $|2x+3|=2$ (c) $|x+3|<2$ (d) $|x+3|-1>4$

10. (**6** Pts) Solve the following inequalities.
    (a) $4+2x\leq 7-3x$ (b) $-4\leq -3x+5<2$

11. (**8** Pts) Find the inverse of the functions (a) $f(x)=\frac{x+3}{2x+5}$ (b) $g(x)=4x+3$

12. (**6** Pts) Starting with the basic function, sketch the graph of the function $f(x)=-(x-2)^2-3$ by various transformations in order.

13. (**10** Pts) From the given graph of the function $y=f(x)$, make a hand-drawn graph of the following functions. (a) $y=f(x)+2$ (b) $y=f(x-4)$ (c) $y=-\frac{1}{2}f(x)$ (d) $y=-f(x-4)+2$



Andrew Short

1) a) $-2(3x+2)-4=2(x-1)+3x$

$-6x-4-4=2x-2+3x$

$-6x-8=5x-2$

$+6x \qquad +6x$

$-8=11x-2$

$+2 \qquad +2$

$\frac{-6}{11}=\frac{11x}{11}$

$x=-6/11$

b) $x^2+7x+12=0$

$(x+3)(x+4)=0$

$x=-3,\ x=-4$

-12, -4, 3, 1

c) $2x^2-x-6=0$

$(2x^2-4x)(3x-6)$

$2x(x-2)3(x-2)$

$(2x+3)(x-2)$

$x=2,\ x=-3/2$

d) $4x^2-9=0$

$(2x+3)(2x-3)=0$

$x=-3/2,\ x=3/2$

2)

a) $x^2=u$

$x^4-10x^2+9=0$

$u^2-10u+9=0$

$(u-9)(u-1)$

$u=9,\ u=1$

$\sqrt{x^2}=\sqrt{9},\ \sqrt{x^2}=\sqrt{1}$

$x=\pm 3,\ x=\pm 1$

b) $(x^3-2x^2)-(4x+8)=0$

$x^2(x-2)-4(x-2)=0$

$(x^2-4)(x-2)=0$

$(x+2)(x-2)(x-2)=0$

$x=-2,\ x=2$

c) $\sqrt[2]{x+6}=x^2$

-6, 2, -3, -1

$x+6=x^2$

$x^2-x-6=0$

$(x-3)(x+2)$

$x=3$ $\cancel{x=-2}$

3.) a) $\frac{2x+1}{x+2} - \frac{x-4}{x^2+5x+6}$

$\frac{6}{2}\ \frac{3}{5}$

$\frac{x+3}{x+3} \cdot \frac{2x+1}{x+2} - \frac{x-4}{(x+2)(x+3)}$

$\frac{2x^2+x+6x+3-x+4}{(x+2)(x+3)} = \frac{2x^2+6x+7}{(x+2)(x+3)}$

$\frac{-4}{+5}$

b) $\frac{x^2-5x-6}{x^2+5x+6} \div \frac{x^2-x-6}{x^2-4}$

$\frac{(x-6)(x+1)}{(x+3)\cancel{(x+2)}} \cdot \frac{\cancel{(x+2)}(x-2)}{(x-3)(x+2)}$

$\frac{(x-6)(x+1)(x-2)}{(x+3)(x-3)(x+2)}$

4.) $[0,5)$ $[4,5]$

$\frac{25-5}{4-0} = \frac{20}{4} = 5$

Average Rate of Change = 5

5.) a) $(3-3i)-(4-2i)$

$(3-3i)+(-4+2i)$

$(3-4)+(-3i+2i)$

$-1-i$

b) $(3-3i)(4-2i)$

$12-6i-12i+6i^2$

$12-18i+6(-1)$

$12-18i-6$

$6-18i$

c) $\frac{3-3i}{4-2i} \cdot \frac{4+2i}{4+2i}$

$\frac{(3-3i)(4+2i)}{16-4i^2}$

$\frac{18-6i}{20}$

$\frac{18}{20} - \frac{6}{20}i$

$\frac{9}{10} - \frac{3}{10}i$

6) a) $3x-2=0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

$x \neq 2/3$

$D=(-\infty, 2/3) \cup (2/3, \infty)$

b) $g(x)=\sqrt{2x-5}$

$D=[5/2, \infty)$

7) a) $3x-4y=1$

$3x = 1+4y$

$\frac{3x}{4} - \frac{1}{4} = \frac{4y}{4}$

$\frac{3}{4}x - 1/4 = y$

$m = 3/4$

$3 = 3/4(-2) + b$

$3 = -6/4 + b$

$+6/4 \quad +6/4$

$9/2 = b$

$y = 3/4x + 9/2$

7) b) $m = -4/3$

$-2 = -4/3(1) + b$

$4/3 \quad 4/3$

$-2/3 = b$

$y = -4/3x - 2/3$

8) a) $(f+g)(3) = f(3) + g(3)$

$f(3) = 3^2 + 3(3) - 4$

$f(3) = 9 + 9 - 4$

$f(3) = 14$

$g(3) = \frac{2(3)+3}{3-2}$

$g(3) = \frac{6+3}{1}$

$g(3) = 9$

$(f+g)(3) = 9 + 14 = 23$

b) $(f \cdot g)(x) = f(x) \cdot g(x)$

$(x^2+3x-4)\left(\frac{2x+3}{x-2}\right)$

8) c) $(f \circ g)(x)$

$f(g(x))$

$$\left(\frac{2x+3}{x-2}\right)^2 + 3\left(\frac{2x+3}{x-2}\right) - 4$$

d) $(g \circ f)(x)$

$g(f(x))$

$$\frac{2(x^2+3x-4)+3}{(x^2+3x-4)-2}$$

e) $(f \circ f)(2)$

$f(f(2))$

$f(2) = 2^2 + 3(2) - 4$

$f(2) = 4 + 6 - 4$

$f(2) = 10 - 4$

$f(2) = 6$

$f(6) = (6)^2 + 3(6) - 4$

$f(6) = 36 + 18 - 4$

$f(6) = 54 - 4$

$f(6) = 50$

$(f \circ f)(2) = 50$

9) a) $|x| = 2$

$|x| = \pm 2$

b) $|2x+3| = 2$

$2x + 3 = 2$
$-3 \quad -3$
$\frac{2x}{2} = \frac{-1}{2}$
$x = -1/2$

$2x + 3 = -2$
$-3 \quad -3$
$\frac{2x}{2} = \frac{-5}{2}$
$x = -5/2$

$|2x+3| = 2$

$x = -1/2, \; x = -5/2$

c) $|x+3| < 2$

$x + 3 < 2$
$-3 \quad -3$
$x < -1$

$x + 3 < -2$
-3
$x < -5$

$D = (-5, -1)$

d) $|x+3| - 1 > 4$
$+1 \quad +1$

$|x+3| > 5$

$x + 3 < -5$
-3
$x < -8$

$x + 3 > 5$
$x > 2$

$D = (-\infty, -8) \cup (2, \infty)$

10) a) $4+2x \le 7-3x$
$\quad +3x \qquad +3x$

$4+5x \le 7$
$-4 \qquad -4$

$\frac{5x}{5} \le \frac{3}{5}$

$x \le 3/5$

$(-\infty, \frac{3}{5}]$

b) $-4 \le -3x+5 < 2$
$-5 \qquad -5 \quad -5$

$\frac{-9}{-3} \le \frac{-3x}{-3} < \frac{-3}{-3}$

$3 \ge x > 1$

$(1, 3]$

11) a) $f(x) = \frac{x+3}{2x+5}$

$(2y+5)\, x = \frac{y+3}{\cancel{(2y+5)}} \cancel{(2y+5)}$

$x(2y+5) = y+3$

$2xy+5x = y+3$

$2xy - y = -5x+3$

$(2x-1)y = -5x+3$

$y = \frac{-5x+3}{2x-1} = f^{-1}(x)$

11) b) $g(x) = 4x+3$

$x = 4y + 3$
$-3 \qquad -3$

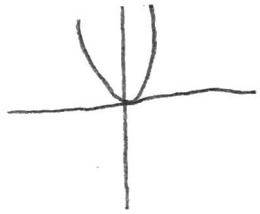
$\frac{x}{4} - \frac{3}{4} = \frac{4y}{4}$

$y = \frac{x}{4} - 3/4$

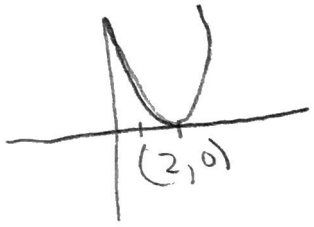
$g(x)^{-1} = y = x/4 - 3/4$

12) $f(x) = -(x-2)^2 - 3$

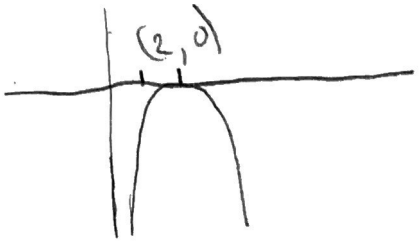
a) $f(x) = x^2$



b) $f(x) = (x-2)^2$

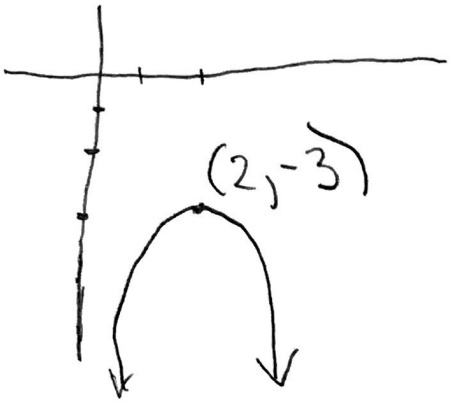


c) $f(x) = -(x-2)^2$

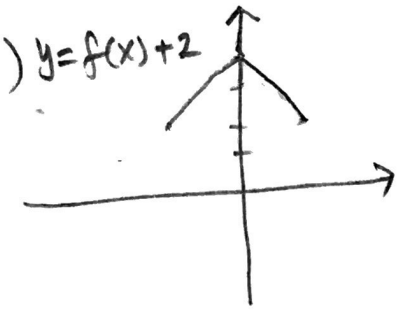


least graph on back

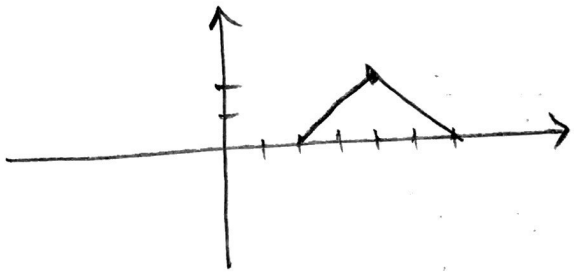
12) d) $f(x) = -(x-2)^2 - 3$



13. (a) $y = f(x) + 2$

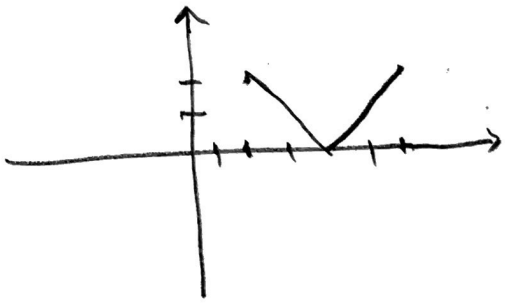


(b) $y = f(x-4)$



(c) $y = -\frac{1}{2} f(x)$

(d) $y = -f(x-4) + 2$

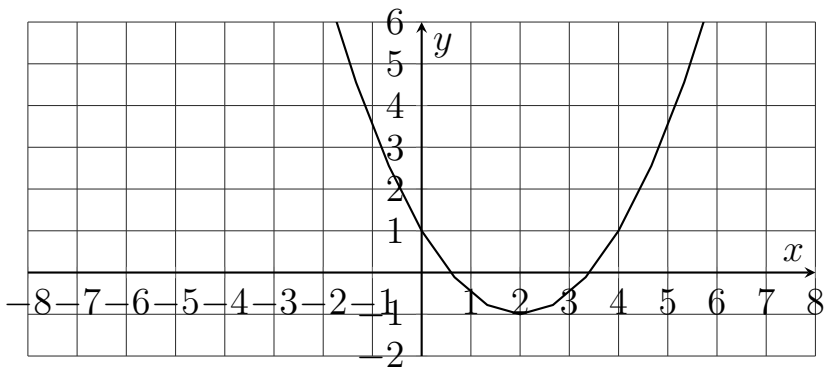


Exam 2 MTH 132 Spring 2023 Total Pts:100 3/3/2023

Show all work for full credit.

1. **(No Calculator)** For the quadratic function $f(x) = x^2 - 4x - 5$ (8 Pts)
   (a) find the vertex and the axis of symmetry
   (b) find the x- and y-intercepts
   (c) sketch the graph of the parabola f
   (d) find the maximum or minimum value of $f(x)$
   (e) find the range of f and the intervals where f is increasing/decreasing

2. Write the equation for the graphed function. (6 Pts)



3. Use the synthetic division to find the quotient and remainder: (6 Pts)
   $(2x^4 - 3x^3 - x^2 + 2x - 7) \div (x - 2)$. Is $(x - 2)$ a factor?

4. **(No Calculator)** For the polynomial $f(x) = -2(x+1)^2(x-1)^3(x-3)^2(x-4)$, (10 Pts)
   (a) find the leading term and describe the end behavior of f,
   (b) list all the zeros with multiplicities,
   (c) mention whether the graph crosses or touches at each of the zeroes,
   (d) sketch the graph of f using (a)-(c).

5. **(No Calculator)** For the polynomial function $f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$
   (a) list all potential rational zeros,
   (b) find two actual zeros by finding the value of f at numbers from (a)
   (c) use the synthetic division twice to factorize f completely
   (d) find all real zeros with their multiplicities and determine the touching or crossing
   (e) determine the end behavior and sketch the graph of $f(x)$ (12 Pts)

6. Use the given information about the polynomial graph to write the equation. (6 Pts)
   degree 3, zeros at $x = 3, x = -1$, and $x = 2$, y-intercept at (0,-4)

7. **(No Calculator)** For the rational function $f(x) = \frac{x-1}{(x-3)(x+1)}$ (10 Pts)
   (a) find the horizontal and vertical asymptotes
   (b) find x-intercept(s) and y-intercept
   (c) use (a), (b) and the analysis described in the class to sketch the graph of $f(x)$

8. An account is opened with an initial deposit of $\$10,000$ and earns 8% interest compounded quarterly. What will the account be worth in 10 years? How long will it take the initial amount to be double? (6 Pts)

9. Use the transformation to sketch the graph: (6 Pts)
   (a) $f(x) = 2^{x-1} + 3$ (b) $g(x) = -\ln(x-3)$

10. Find the exact value of each logarithm without using a **calculator**. (6 Pts)
    (a) $\log_2 32$ (b) $\log_5 \frac{1}{125}$ (c) $\log_7 343$ (d) $\log 1000$ (e) $\ln \sqrt[4]{e}$ (f) $\log 0.1$

11. Solve the following equations. (24 Pts)
    (a) $5^{2x-3} = 125$
    (b) $\log_2(2x+3) = 3$
    (c) $\log_3(3x-5) = \log_3(x+3)$
    (d) $3^{x-3} + 2 = 8$
    (e) $\log_6 x + \log_6(x-1) = 1$
    (f) $\log(x+8) - \log(x-1) = 1$

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03/03/23

1

a) $h = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$k = f(2) = 2^2 - 4(2) - 5 = -9$

$V(2,-9)$

Axis of Symmetry: $x=2$

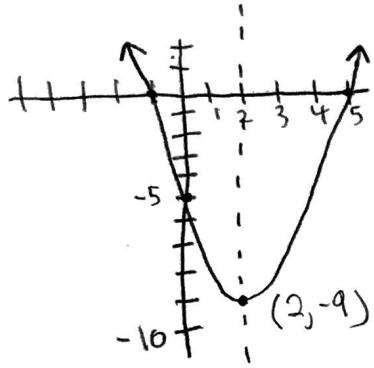
b) x-int $= x^2 - 4x - 5$

$(x+1)(x-5)$

$x=-1$
$x=5$

y-int $= -5$

c)



d) maximum: none

minimum: $(2,-9)$

e) Range: $[-9, \infty)$

Increasing: $(2, \infty)$

Decreasing: $(-\infty, 2)$

2

Vertex: $(2,-1)$

$f(x) = a(x-h)^2 + k$

$= a(x-2)^2 - 1$

$1 = a(0-2)^2 - 1$

$1 = 4a - 1$

$4a = 2$

$a = \frac{2}{4} = \frac{1}{2}$

$f(x) = \frac{1}{2}(x-2)^2 - 1$

3 $(2x^4 - 3x^3 - x^2 + 2x - 7) \div (x-2)$

2	2	-3	-1	2	-7
	2	4	2	2	8
	2	1	1	4	1 = R

$(2x^3 + x^2 + x + 4)$ $R=1$

$(x-2)$ is not a factor

Turn Page →

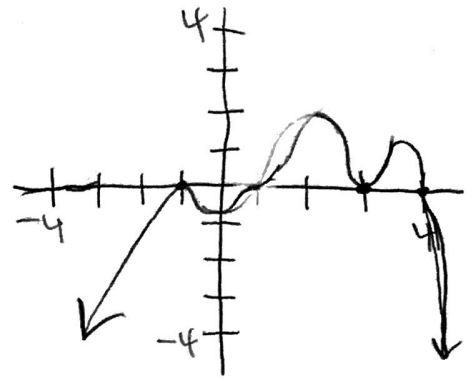
$f(x) = -2(x+1)^2(x-1)^3(x-3)^2(x-4)$

4

a) Leading term: $-2x^8$ End Behavior:

b+c) $x=-1$ mult 2 - touches
$x=1$ mult 3 - crosses
$x=3$ mult 2 - touches
$x=4$ mult 1 - crosses

d)



5

$f(x) = x^4 - 6x^3 + 9x^2 + 4x - 12$

a) $\dfrac{\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1}{\pm 1} = \pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$

b) $f(1) = 1 - 6 + 9 + 4 - 12 \neq 0$
$f(-1) = 1 + 6 + 9 - 4 - 12 = 0$ ✓
$f(2) = 16 - 48 + 36 + 8 - 12 = 0$
$f(3) = 81 - 162 + 81 + 12 - 12 = 0$

$x=2$

$x=3$

c)

2	1	-6	9	4	-12
		2	-8	2	12
3	1	-4	1	6	$0 = R$
		3	-3	-6	
	1	-1	-2	$0 = R$	

$x^2 - x - 2$

$(x-2)(x+1)$

d) $(x-2)(x+1)(x-2)(x-3)$

$(x-2)^2(x+1)(x-3)$

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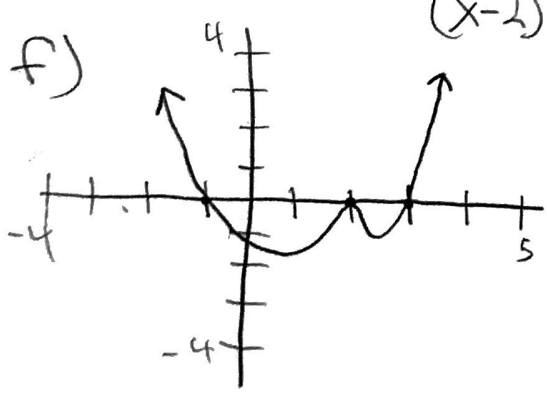
f)



End Behavior

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(6) $f(x) = a(x-3)(x+1)(x-2)$

$-4 = a(0-3)(0+1)(0-2)$

$-4 = 6a$

$a = -\frac{4}{6} = -\frac{2}{3}$

$f(x) = -\frac{2}{3}(x-3)(x+1)(x-2)$

(7) a) vertical: $x=3,\ x=-1$

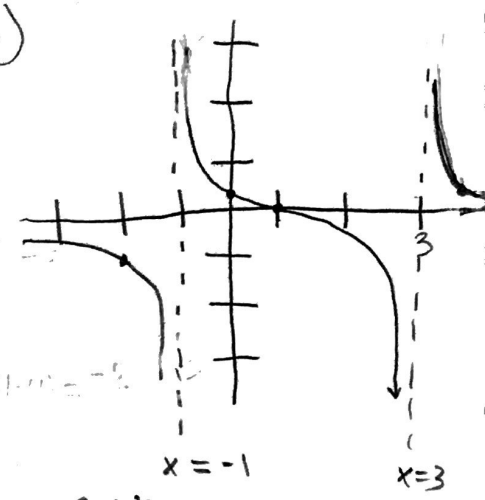
horizontal: $y=0$

b) x-int: $0 = \frac{x-1}{(x-3)(x+1)}$ $x=1$

y-int: $f(0) = \frac{0-1}{(0-3)(0+1)} = \frac{-1}{-3} = \frac{1}{3}$

$f(-2) = -\frac{3}{5},\ f(4) = \frac{3}{5}$

c)



(8) $A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 10{,}000\left(1+\frac{0.08}{4}\right)^{4(10)}$

$A = \$\ 22{,}080.40$

$20000 = 10000\left(1+\frac{0.08}{4}\right)^{4t}$

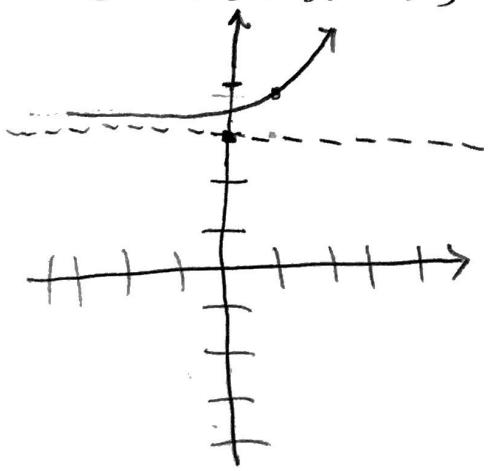
$2 = (1.02)^{4t}$

$\ln 2 = \ln (1.02)^{4t}$

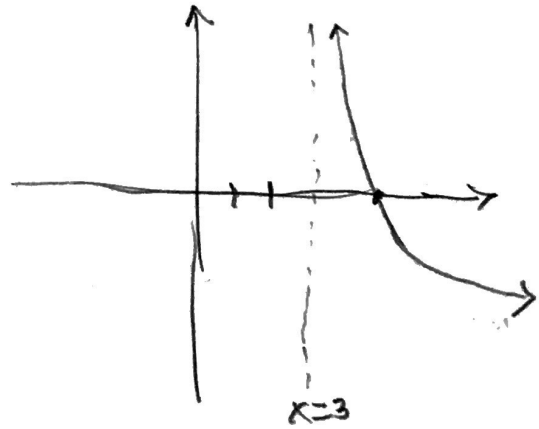
$\ln 2 = (4t)\ln(1.02)$

$t = \frac{\ln 2}{4\ln(1.02)} \approx 8.75$ years

(9) a) $f(x) = 2^{x-1}+3$



b) $g(x) = -\ln(x-3)$



(10)

a) $\log_2 32$

5

b) $\log_5 \frac{1}{125}$

-3

c) $\log_7 343$

3

d) $\log 1000$

3

e) $\ln \sqrt[4]{e}$

$\frac{1}{4}$

f) $\log 0.1$

-1

(11)

a) $5^{2x-3} = 125$

$5^{2x-3} = 5^3$

$2x-3 = 3$

$2x = 6$

$x = 3$

b) ${}_2\log_2(2x+3) = {}_2 3$

$2x+3 = 8$

$2x = 5$

$x = \frac{5}{2}$

c) ${}_3\log_3(3x-5) = {}_3\log_3(x+3)$

$3x-5 = x+3$

$2x = 8$

$x = 4$

d) $3^{x-3} + 2 = 8$

$\ln 3^{x-3} = \ln 6$

$(x-3)\ln 3 = \ln 6$

$x-3 = \frac{\ln 6}{\ln 3}$

$x = \frac{\ln 6}{\ln 3} + 3$

e) $\log_6 x + \log_6(x-1) = 1$

${}_6\log_6(x^2-x) = {}_6 1$

$x^2 - x = 6$ $\quad \cancel{x = -2}$

$x^2 - x - 6 = 0$ $\quad x = 3$

$(x+2)(x-3) = 0$

f) $\log(x+8) - \log(x-1) = 1$

$\log \frac{x+8}{x-1} = 1$ $\quad 9x = 18$

$\frac{x+8}{x-1} = 10$ $\quad x = 2$

$x+8 = 10x - 10$

Exam 3 MTH 132 Spring 2023 Total Pts:100 3/21/2023

Name:_____ Total Received:

Show all work for full credit. Write all your solutions on the given blank papers.

1. Find radius r if $\alpha = 45°$ and arc length is $s = 20$ in. (4 Pts)

2. Find a, b, and angle α if $\beta = 30°$ and $c = 12$ in for a right triangle. (5 Pts)

3. Convert the following. (4 Pts)
   (a) $330°$, $460°$ (into radian)
   (b) $\frac{3\pi}{7}$, $\frac{2\pi}{9}$ (into degree)

4. Name the quadrant containing the terminal side of α. (4 Pts)
   (a) $\sin\alpha < 0$ and $\tan\alpha > 0$
   (b) $\cos\alpha > 0$ and $\sin\alpha < 0$

5. Evaluate each expression. (6 Pts)
   (a) $\cos^{-1}(\frac{\sqrt{3}}{2})$ (b) $\tan^{-1}(-1)$ (c) $\cos(\sin^{-1}(\frac{5}{8}))$

6. Find the exact value of each of the other five trigonometric functions for the angle θ given that $\sin\theta = \frac{3}{5}$ and θ is in quadrant II. (6 Pts)

7. To find the height of a tree, a person walks to a point 40 feet from the base of the tree. She measures an angle of $4°$ between a line of sight to the top of the tree and the ground. Find the height of the tree. (Draw a diagram.) (5 Pts)

8. What is the linear velocity in miles per hour of the tip of a 20-inch lawnmower blade that is rotating at 2200 revolutions per minutes? (1 mile=5280 ft) (5 Pts)

9. (i) Sketch the following angles in their standard positions and then find the reference angle for $\frac{7\pi}{6}$, $\frac{3\pi}{4}$, $-150°$, $210°$, $315°$, $\frac{2\pi}{3}$.
   (ii) Use reference angle to find the exact value of each trigonometric functions (No Calculator).
   (a) $\cos\frac{7\pi}{6}$ (b) $\tan\frac{3\pi}{4}$ (c) $\cos(-150°)$ (d) $\cot 210°$ (e) $\cos(315°)$ (f) $\sec(\frac{2\pi}{3})$ (15 Pts)

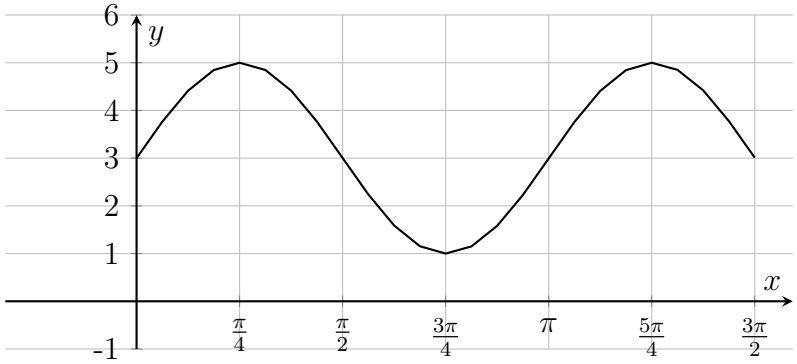
10. Complete the following table. (5 Pts)

	$0°/0$	$30°/\frac{\pi}{6}$	$45°/\frac{\pi}{4}$	$60°/\frac{\pi}{3}$	$90°/\frac{\pi}{2}$
sin					
cos					
tan					

See next page for more questions.

11. State the amplitude, period, midline, and phase shift for each equation, and graph one cycle. List starting and ending points on x-axis. (15 Pt)
(a) $y = 2\sin[2(x - \frac{\pi}{4})] + 3$ (b) $y = -3\cos(2x) - 4$ (c) $f(x) = -\tan(x - \frac{\pi}{4})$

12. Find A, B, D and then write the equation for the graph in the form of
(i) $y = A\sin[B(x - C)] + D$ and (ii) $y = A\cos[B(x - C)] + D$ (8 Pts)



13. Write reciprocal identities, quotient identities, Pythagorean identities. (6 Pts)

14. Verify the identities. (12 Pts)
(a) $\frac{\cos^3 x + \cos x \sin^2 x}{\sin x} = \cot x$, (b) $\frac{1}{1-\sin x} = \sec^2 x + \tan x \sec x$,
(c) $\cot x + \tan x = \sec x \csc x$, (d) $\frac{\cot x + 1}{\csc x} = \cos x + \sin x$

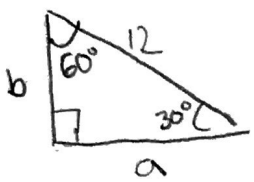
Andrew Short

1.) $s = r\theta$

$\left(\frac{4}{\pi}\right) 20 = r\left(\frac{\pi}{4}\right)\cdot\left(\frac{4}{\pi}\right)$

$\frac{80}{\pi}\text{ in} = r$

2.)



$\sin 30° = \frac{b}{12}$

$\frac{1}{2} = \frac{b}{12}$

$b = 6$

$\cos 30° = \frac{a}{12}$

$\frac{\sqrt{3}}{2} = \frac{a}{12}$

$\frac{6\sqrt{3}}{12} = \frac{a}{12}$

$a = 6\sqrt{3}$

$\alpha = 60°$

3.) a) $330 \cdot \frac{\pi}{180} = \frac{330\pi}{180} = \frac{11\pi}{6}$

$460 \cdot \frac{\pi}{180} = \frac{23\pi}{9}$

b) $\frac{3\pi}{7} \cdot \frac{180}{\pi} = 77.1°$

$\frac{2\pi}{9} \cdot \frac{180}{\pi} = 40°$

4.)

a) Q III

b) Q IV

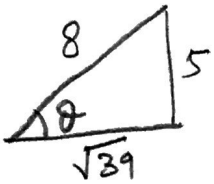
5.) a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$

b) $\tan^{-1}(-1) = -45°$

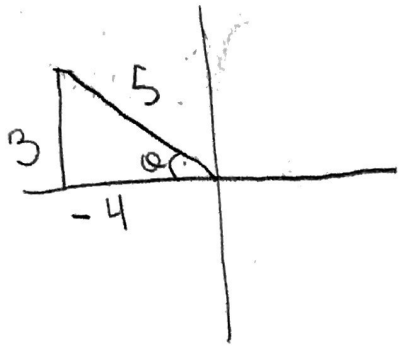
c) $\cos\left(\sin^{-1}\left(\frac{5}{8}\right)\right) = \cos\theta = \frac{\sqrt{39}}{8}$

$\theta = \sin^{-1}\left(\frac{5}{8}\right)$

$\sin\theta = \frac{5}{8} = \frac{o}{h}$



6.)

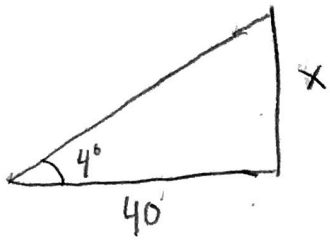


$\sin\alpha = 3/5$ $\csc\alpha = 5/3$

$\cos\alpha = -4/5$ $\sec\alpha = -5/4$

$\tan\alpha = -3/4$ $\cot\alpha = -4/3$

7.)



$\tan(4°) = \frac{x}{40}$

$x = \tan(4°)(40)$

$x = 2.8$ ft

8.) $w = 2200$ rpm $= 2200 \times 2\pi$ rad/min

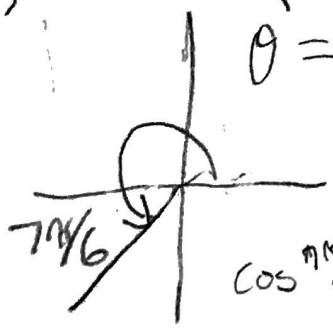
$v = rw = 10 \times 2200 \times 2\pi \ \frac{in}{min}$

$= 10 \times 2200 \times 2\pi \ \frac{in}{min} \times \frac{1 ft}{12 in} \times \frac{1 mile}{5280 ft}$

$= \frac{10 \times 2200 \times 2\pi}{12 \times 5280} \ \frac{mile}{min} \times \frac{60 min}{1 hr}$

$= 130.9$ miles/hr

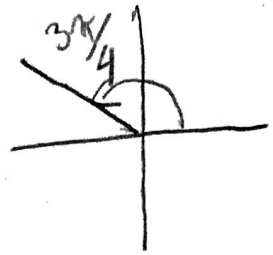
9.)



$\theta' = \pi/6$

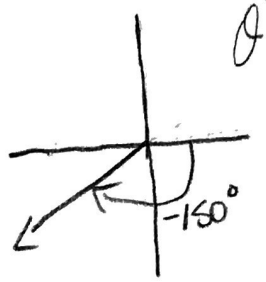
$\cos 7\pi/6 = -\sqrt{3}/2$

9.)



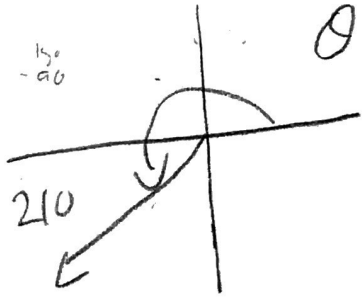
$\theta' = \pi/4$

$\tan 3\pi/4 = -1$



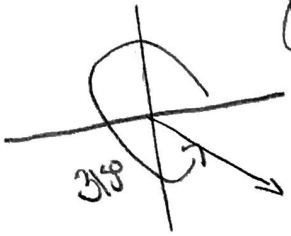
$\theta' = 30°$

$\cos(-150°) = -\frac{\sqrt{3}}{2}$



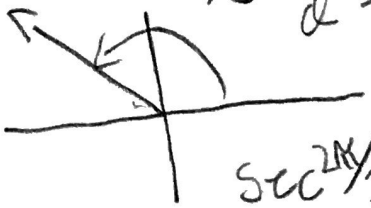
$\theta' = 30°$

$\cot 210 = \frac{3}{\sqrt{3}}$



$\theta' = 45°$

$\cos(315°) = \frac{\sqrt{2}}{2}$



$\theta' = \pi/3$

$\sec 2\pi/3 = -2$

Andrew Short

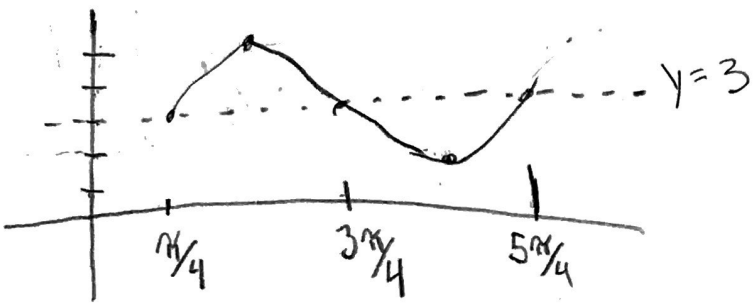
11. $y = 2\sin[2(x - \pi/4)] + 3$

a) Amp: 2

Period: $\frac{2\pi}{2} = \pi$

midline: $y = 3$

Phase shift: $\pi/4$



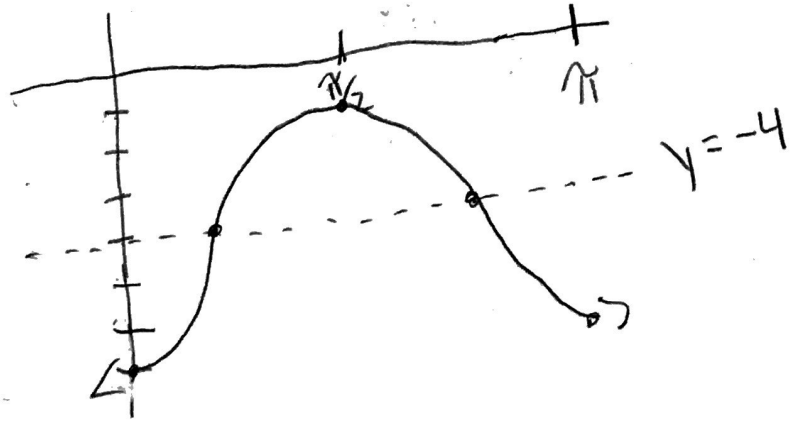
b) $y = -3\cos(2x) - 4$

Amp: 3

Period: $\frac{2\pi}{2} = \pi$

Midline: $y = -4$

Phase shift: 0



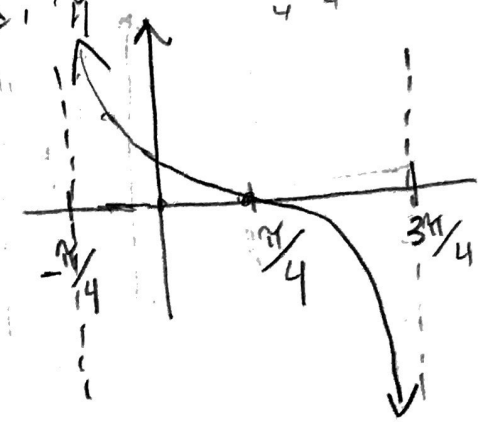
c) $f(x) = -\tan\left(x - \frac{\pi}{4}\right)$

right $\pi/4$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\frac{\pi}{4}$ $\frac{\pi}{4}$

period: π



12.)

$A = 2$

$B = 2$

$D = 3$

i) $y = 2\sin(2x) + 3$

ii) $y = 2\cos\left[2\left(x - \frac{\pi}{4}\right)\right] + 3$

13.

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\csc^2\theta - \cot^2\theta = 1$

$\csc^2\theta = 1 + \cot^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

14.

a) $\frac{\cos^3 x + \cos x \sin^2 x}{\sin x} = \cot x$

LS: $\frac{\cos x(\cos^2 x + \sin^2 x)}{\sin x}$

$\frac{\cos x}{\sin x} = \cot x = RS$

Andrew Short

14.

b) $\frac{1}{1-\sin x} = \sec^2 x + \tan x \sec x$

RS: $\sec^2 x + \tan x \sec x$

$\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$= \frac{1+\sin x}{\cos^2 x} = \frac{1+\sin x}{1-\sin^2 x} = \frac{\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(1-\sin x)} = \frac{1}{1-\sin x} = LS$

c) $\cot x + \tan x = \sec x \csc x$

LS: $\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \sec x \csc x = RS$

d) $\frac{\cot x + 1}{\csc x} = \cos x + \sin x$

LS:

$\left(\frac{\cos x}{\sin x} + 1\right)(\sin x)$

$\frac{\cos x(\sin x)}{\sin x} + \sin x = \cos x + \sin x = RS$

Final Exam MTH 132 Spring 2023 Total Pts: 100 4/25/2023

Show all work for full credit.
NO CALCULATOR for the problems which require EXACT VALUES for answers.

1. Find the exact value of the indicated expression. (9 Pts)
   (a) $\sin 75°$ (b) $\cos 70° \cos 10° + \sin 70° \sin 10°$ (c) $\sin 22.5°$

2. Find the exact values without using a calculator. (8 Pts)
   (a) $\cos\left(\sin^{-1}(\frac{4}{7})\right)$, (b) $\cos\left(\sin^{-1}(\frac{2}{3}) + \cos^{-1}(\frac{3}{4})\right)$

3. Find the exact values of $\sin(\alpha+\beta), \cos(\alpha+\beta)$ and $\tan(\alpha+\beta)$ given that $\sin\alpha = -\frac{5}{7}$ and $\sin\beta = \frac{12}{13}$, with α in quadrant III and β in quadrant II. (7 Pts)

4. Find the exact value of $\sin(2\theta)$, $\cos(\frac{\theta}{2})$, and $\tan(2\theta)$ given that $\sin\theta = \frac{4}{5}$, $\frac{\pi}{2} < \theta < \pi$. (7 Pts)

5. Find the exact solutions over the indicated intervals. (12 Pts)
   (a) $2\sin\theta - \sqrt{3} = 0$, all solutions
   (b) $\cos(2\theta) = \frac{1}{2}$, $0° \le x < 360°$
   (c) $2\sin^2\theta - \sin\theta - 1 = 0$, all solutions

6. Solve each triangle using the law of sines or the law of cosines (or both). (12 Pts)
   (a) $B = 40°,\ C = 70°,\ b = 7$
   (b) $C = 70°,\ a = 5ft,\ b = 8ft$
   (c) $a = 3,\ b = 5,\ c = 7$

7. Find the area of each triangle. (5 Pts)
   $B = 35°,\ a = 6,\ b = 4$

8. Consider two complex numbers $z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ z_2 = 1 + i$. (8 Pts)
   (a) Change them to polar form $r(\cos\theta + i\sin\theta)$.
   (b) Use the polar forms of z_1 and z_2 to find $z_1 z_2$ and $(z_1)^{13} = (-\frac{1}{2} + \frac{\sqrt{3}}{2}i)^{13}$.

9. For the vectors $\overrightarrow{u} = \langle -3, 2\rangle$, $\overrightarrow{v} = \langle 4, 2\rangle$, (8 Pts)
   (a) find $2\overrightarrow{u} - 3\overrightarrow{v}$, $|\overrightarrow{u}|$, $|\overrightarrow{v}|$, $\overrightarrow{u}\cdot\overrightarrow{v}$,
   (b) find the angle between the vectors $\overrightarrow{u}$ and $\overrightarrow{v}$

10. There were 130 faculty at a conference. If there were 18 more women than men attending, how many of each gender attended the conference? (6 Pts)

11. Solve the following system of linear equations by elimination. (8 Pts)

$$\begin{cases} x + y + z = 1 \\ 2x - 3y - 2z = 4 \\ -3x + 2y + 4z = 1 \end{cases}$$

12. Use paper and pencil to evaluate (if defined) AD, BC, $3A - 2C$ for the matrices

$$A = \begin{bmatrix} -3 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix},\ B = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix},\ C = \begin{bmatrix} -2 & 1 & 2 \\ 3 & 1 & 4 \end{bmatrix},\ D = \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 1 & 1 \end{bmatrix}.$$ (8 Pts)

Formulas: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$, $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$, $\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$