

Name: _____

Total Received:

Show all work for full credit. Write all your solutions on the blank papers.

1. Find the domain of functions (a) $f(x) = \sqrt{2x - 3}$, (b) $g(x) = \frac{x+2}{x^2-1}$ (5 Pts)

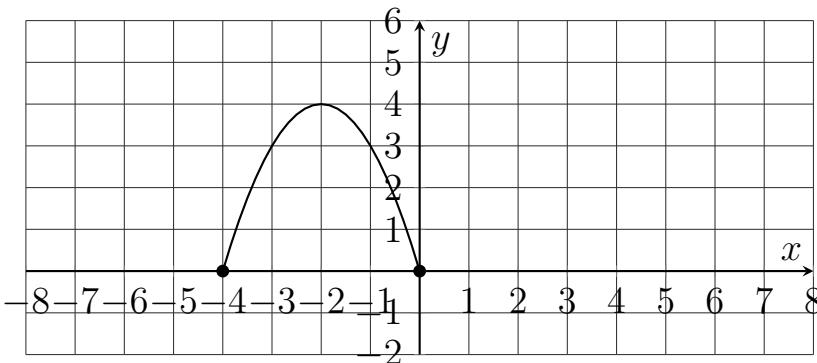
2. Find the functions f/g , $f \circ g$, and $g \circ f$ for the functions

$$f(x) = 2x^2 + x - 3, \quad g(x) = \frac{\sqrt{2x-5}}{x+3} \quad (5 \text{ Pts})$$

3. Find an equation of the line through the point (2,-1) and is perpendicular to the line $3x - 2y = 7$. (5 Pts)

4. From the given graph of the function $y = f(x)$, make a hand-drawn graph of the following functions. (6 Pts)

(a) $y = f(x) + 2$ (b) $y = f(x - 2)$ (c) $y = \frac{1}{4}f(x)$ (d) $y = -f(x + 2)$



5. Find the inverse of the functions (a) $f(x) = \frac{x+3}{2x-1}$, (b) $g(x) = -3x^3 + 2$. (7 Pts)

6. Solve the logarithmic equation $\log_2(x + 2) + \log_2(x - 1) = 2$. (5 Pts)

7. Find the limit numerically by evaluating limit from the Left and limit from the Right for the limit $\lim_{x \rightarrow 1} \frac{\ln x}{x-1}$. (5 Pts)

8. Evaluate the limit **algebraically**, if it exists. State one-sided limits if infinite. (15 Pts)

(a) $\lim_{x \rightarrow 3} \frac{x^2-9}{x^2-x-6}$ (b) $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2}$ (c) $\lim_{h \rightarrow 0} \frac{(3+h)^2-9}{h}$

(d) $\lim_{x \rightarrow 1} \frac{x+4}{x-1}$ (e) $\lim_{x \rightarrow \frac{1}{2}} \frac{2x^2-3x+1}{2x^2+9x-5}$

9. Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f . (6 Pts)

$$f(x) = \begin{cases} x-1 & \text{if } x < 0 \\ x^2-1 & \text{if } 0 \leq x < 3 \\ 2x-1 & \text{if } x \geq 3 \end{cases}$$

10. Calculate the limit at ∞ and $-\infty$ and find the horizontal asymptote(s). (10 Pts)
- (a) $\lim_{x \rightarrow \infty} \frac{-2x^2+x-3}{5x-3x^2+4}$, $\lim_{x \rightarrow -\infty} \frac{-2x^2+x-3}{5x-3x^2+4}$ (b) $\lim_{x \rightarrow \infty} \frac{x^2+x+4}{x^4-2x+1}$, $\lim_{x \rightarrow -\infty} \frac{x^2+x+4}{x^4-2x+1}$
- (c) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^4+x}}{3x^2+4}$, $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^4+x}}{3x^2+4}$ (d) $\lim_{x \rightarrow \infty} \frac{x^3-x}{x+4}$, $\lim_{x \rightarrow -\infty} \frac{x^3-x}{x+4}$

11. Sketch the graph of an example of a function f that satisfies all of the given conditions.

State the equations of vertical and horizontal **asymptotes**. (7 Pts)

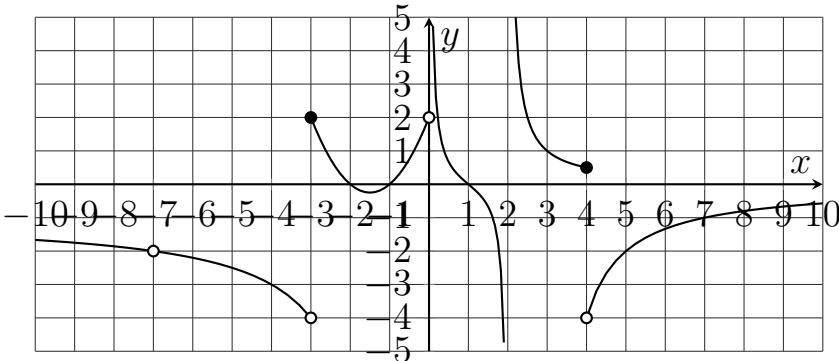
$$\begin{aligned}\lim_{x \rightarrow \infty} f(x) &= 1 & \lim_{x \rightarrow -\infty} f(x) &= 0, & \lim_{x \rightarrow 2} f(x) &= \infty, \\ \lim_{x \rightarrow 0} f(x) &= 0, & f(0) &= 2, & \lim_{x \rightarrow -2^+} f(x) &= \infty, & \lim_{x \rightarrow -2^-} f(x) &= -\infty\end{aligned}$$

12. From the graph of $y = f(x)$ in number 13, find the following. (10 Pts)

$$\begin{array}{llll}(a) \lim_{x \rightarrow -3^+} f(x) & (b) \lim_{x \rightarrow -3^-} f(x) & (c) \lim_{x \rightarrow -3} f(x) & (d) \lim_{x \rightarrow -7} f(x) \\ (e) \lim_{x \rightarrow 0^+} f(x) & (f) \lim_{x \rightarrow 0^-} f(x) & (g) \lim_{x \rightarrow 2} f(x) & (h) \lim_{x \rightarrow 4^+} f(x) \\ (i) \lim_{x \rightarrow \infty} f(x) & (j) \text{vertical and horizontal asymptotes}\end{array}$$

13. From the given graph of $y = f(x)$, find the numbers at which $f(x)$ is discontinuous.

Give reasons for your answers. Check these points for left or right continuity. (7 Pts)



14. State whether the following statements are true (T) or false (F). (7 Pts)

- (a) Rational functions are continuous on $(-\infty, \infty)$.
 (b) The function $f(x) = \tan x$ is continuous at $x = \pi$.
 (c) The limit of a continuous function always exists.
 (d) A polynomial function can have a vertical asymptote.
 (e) If $\lim_{x \rightarrow \infty} f(x) = 2$, then $f(x)$ has a horizontal asymptote.
 (f) If $\lim_{x \rightarrow 3^+} f(x) = f(3)$, then $f(x)$ is continuous from the right.
 (g) The product of two continuous functions may not be a continuous function.

1. a) $f(x) = \sqrt{2x-3}$ $D: [\frac{3}{2}, \infty)$

$$\downarrow$$

$$\frac{2x-3}{2} \geq 0$$

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b) $g(x) = \frac{x+2}{x^2-1}$ $x \neq -1, 1$ $D = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

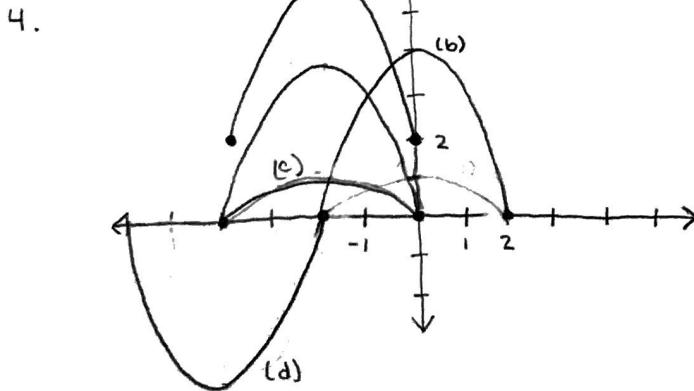
2. $f/g: \frac{2x^2+x-3}{\frac{\sqrt{2x-5}}{x+3}} = \frac{(2x^2+x-3)(x+3)}{\sqrt{2x-5}} = \frac{2x^3+7x^2+9}{\sqrt{2x-5}}$

$$\frac{(2x^3+7x^2+9)}{2x-5} \cdot \frac{(\sqrt{2x-5})}{(\sqrt{2x-5})}$$

$f \circ g: 2\left(\frac{\sqrt{2x-5}}{x+3}\right)^2 + \frac{\sqrt{2x-5}}{x+3} - 3 = \frac{4x-10}{x^2+6x+9} + \frac{\sqrt{2x-5}}{x+3} - 3$

$g \circ f: \frac{\sqrt{2(2x^2+x-3)-5}}{2x^2+x} = \frac{\sqrt{4x^2+2x-11}}{2x^2+x}$

3. $y - y_1 = m(x - x_1)$ $-2y = -3x + 7$
 $y + 1 = m(x - 2)$ $y = \frac{3}{2}x - \frac{7}{2} \rightarrow \text{perpendicular: } x = -\frac{2}{3}$
 $y + 1 = -\frac{2}{3}x + \frac{4}{3}$
 $y = -\frac{2}{3}x + \frac{1}{3}$



$$5. \text{ a) } f(x) = \frac{x+3}{2x-1} \rightarrow x = \frac{y+3}{2y-1} \quad x(2y-1) = y+3 \quad 2xy - x = y+3$$

$$2xy - y = x+3 \quad f^{-1}(x) = \frac{x+3}{2x-1}$$

$$y(2x-1) = x+3$$

$$\text{b) } g(x) = -3x^3 + 2 \rightarrow x = -3y^3 + 2 \quad x-2 = -3y^3$$

$$\frac{x-2}{-3} = y^3 \quad f^{-1}(x) = \sqrt[3]{\frac{x-2}{-3}}$$

$$6. \log_2(x+2) + \log_2(x-1) = 2 \quad \log_2(x+2)(x-1) = 2$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0 \quad \cancel{x=-3} \quad x=2$$

$\lim_{x \rightarrow 1^-} \frac{\ln x}{x-1}$	$\lim_{x \rightarrow 1^+} \frac{\ln x}{x-1}$	$\lim_{x \rightarrow 1^+} \frac{\ln x}{x-1}$	$\lim_{x \rightarrow 1^+} \frac{\ln x}{x-1}$	$\lim_{x \rightarrow 1} \frac{\ln x}{x-1} = 1$
$\begin{array}{ c c } \hline 1^- & \frac{\ln x}{x-1} \\ \hline 0.9 & 1.05 \\ \hline 0.99 & 1.005 \\ \hline 0.999 & 1.005 \\ \hline & \downarrow \\ & 1 \end{array}$	$\begin{array}{ c c } \hline x \rightarrow 1^+ & \frac{\ln x}{x-1} \\ \hline 1.1 & 0.95 \\ \hline 1.01 & 0.995 \\ \hline 1.001 & 0.9995 \\ \hline & \downarrow \\ & 1 \end{array}$			

$$8. \text{ a) } \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)(x+2)} = \lim_{x \rightarrow 3} \frac{x+3}{x+2} = \frac{6}{5}$$

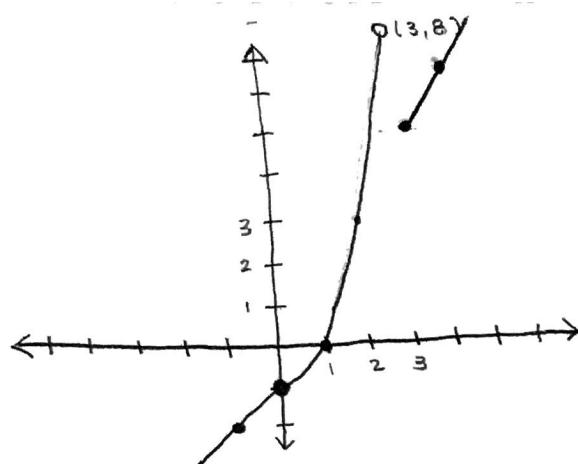
$$\text{b) } \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{(x-4)} = \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = 4$$

$$\text{c) } \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h} = \lim_{h \rightarrow 0} \frac{h^2 + 6h + 9 - 9}{h} = \lim_{h \rightarrow 0} \frac{h(h+6)}{h} = 6$$

$$\text{d) } \lim_{x \rightarrow 1} \frac{x+4}{x-1} \quad 1^+ : \frac{+}{+} = \infty \quad 1^- : \frac{+}{-} = -\infty \quad \text{DNE}$$

$$\text{e) } \lim_{x \rightarrow \frac{1}{2}} \frac{2x^2 - 3x + 1}{2x^2 + 9x - 5} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x-1)(x-1)}{(2x-1)(x+5)} = -\frac{1}{11}$$

9.

check @ $x=0, x=3$ discontinuous @ $x=3$

↓
continuous from
right

10. a) $\lim_{x \rightarrow \infty} \frac{-2x^2}{-3x^2} = \frac{2}{3}$ } HA @ $y = \frac{2}{3}$

$$\lim_{x \rightarrow -\infty} \frac{-2x^2}{-3x^2} = \frac{2}{3}$$

- b) $\lim_{x \rightarrow \infty} \frac{x^2}{x^4} = \frac{1}{x^2} = 0$ } HA @ $y = 0$

$$\lim_{x \rightarrow -\infty} \frac{x^2}{x^4} = \frac{1}{x^2} = 0$$

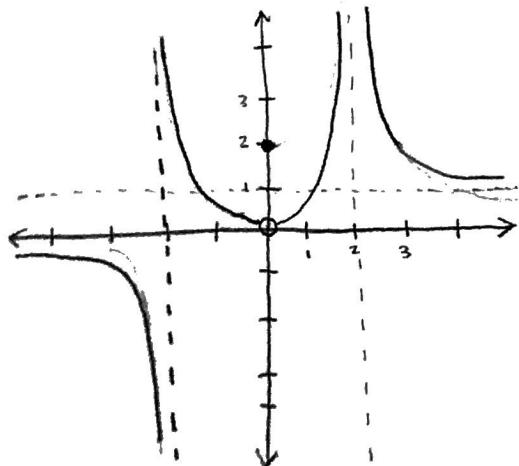
c) $\lim_{x \rightarrow \infty} \frac{2x^2}{3x^2} = \frac{2}{3}$ } HA @ $y = \frac{2}{3}$

$$\lim_{x \rightarrow -\infty} \frac{2x^2}{3x^2} = \frac{2}{3}$$

d) $\lim_{x \rightarrow \infty} \frac{x^3}{x} = x^2 = \infty$ } NO HA

$$\lim_{x \rightarrow -\infty} \frac{x^3}{x} = x^2 = \infty$$

11.

VA: $x = -2, x = 2$ HA: $y = 0, y = 1$

$$12. \text{ a) } \lim_{x \rightarrow -3^+} f(x) = 2$$

$$\text{g) } \lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$\text{b) } \lim_{x \rightarrow -3^-} f(x) = -4$$

$$\text{h) } \lim_{x \rightarrow 4^+} f(x) = -4$$

$$\text{c) } \lim_{x \rightarrow -3} f(x) = \text{DNE}$$

$$\text{i) } \lim_{x \rightarrow \infty} f(x) = 0$$

$$\text{d) } \lim_{x \rightarrow -7} f(x) = -2$$

$$\text{j) VA: } x=0, x=2 \\ \text{HA: } y=-1, y=0$$

$$\text{e) } \lim_{x \rightarrow 0^+} f(x) = \infty$$

$$\text{f) } \lim_{x \rightarrow 0^-} f(x) = 2$$

13. discontinuous: $x = -7$, hole (No Value)
 $x = -3$, jump, continuous from right

$$x = 0, \text{ VA}$$

$$x = 2, \text{ VA}$$

$x = 4$, jump, continuous from left

14. a) F

b) T

c) T

d) F

e) T

f) T

g) F

Name: _____

Total Received:

Show all work for full credit. Do not write anything on question paper except your name.

Do Not use calculator to evaluate the derivatives.

1. Use the **limit definition of the derivative** to compute $f'(x)$ for the function

$f(x) = 2x^2 - 2x + 3$ by computing the following. (8 Pts)

(a) Find $f(x+h) - f(x)$ and simplify.

(b) Find $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$.

(c) Find the slope of the tangent line $m = f'(1)$ at $a = 1$.

(d) Find the equation of the tangent line to the curve $y = f(x)$ at $(1,3)$.

2. Calculate the derivative and simplify whenever possible. ($12 \times 5 = 60$ pts)

(a) $y = \frac{2x+1}{\sin(x^2)+3}$

(b) $y = \cot(\ln(x^2 + 2x))$

(c) $y = \frac{\sec(x+2)}{\ln(\cos x)}$

(d) $y = (\cos(3x) - \cos^3 x)^4$

(e) $y = \tan^3(e^{x^2-1})$

(f) $y = (3x - (3 - 2x^2)^3)^3$

(g) $y = \frac{e^{x^2-2x}}{2x-\tan^{-1}x}$

(h) $y = (x^2 - x) \sin^{-1}(x^2)$

(i) $y = \csc x \ln(1 + \sin x)$

(j) $y = (x^3 - x) \tan^{-1}(x)$

(k) $y = \ln\left(\frac{(x+2)^3}{3+\cos x}\right)$

(l) $y = \tan(\sin(\ln(2x^2 - 4x)))$

3. If $f(0) = 2$, $f'(0) = 3$ and $g(x) = x + e^x$, find the numbers

$(f \cdot g)'(0)$ and $\left(\frac{f}{g}\right)'(0)$. (5 pts)

4. Find the following limits by using $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$. (6 pts)

(a) $\lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}$ (b) $\lim_{x \rightarrow 0} \frac{x^2}{\sin(2x)\sin(3x)}$

5. Find $f''(x)$ for the function $f(x) = xe^{\sin x}$. (5 pts)

6. Find y' for the following implicit functions. (10 pts)

(a) $x \sec y = e^y \sin x$ (b) $x^2 + y^2 = 2x^2y^2$

7. Find $f'(x)$ using the logarithmic differentiation for $f(x) = (\tan x)^x$. (6 pts)

1. a) $f(x) = 2x^2 - 2x + 3$

$$\begin{aligned}f(x+h) &= 2(x+h)^2 - 2(x+h) + 3 - (2x^2 - 2x + 3) \\&= \cancel{2x^2} + 4xh + 2h^2 - \cancel{2x} - 2h + \cancel{3} - \cancel{2x^2} + \cancel{2x} - \cancel{3} \\&= 4xh + 2h^2 - 2h\end{aligned}$$

b) $f'(x) = \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 2h}{h} = \lim_{h \rightarrow 0} (4x - 2 + 2h) = 4x - 2$

c) $m = f'(1)$ at $a = 1$

$$4(1) - 2 = 2$$

d) $y = f(x)$ at $(1, 3)$

$$3 = 2(1) + b \rightarrow 1 = b$$

$$y = 2x + 1$$

2. a) $y = \frac{2x+1}{\sin(x^2) + 3}$

$$y' = \frac{(2)(\sin(x^2) + 3) - (2x+1)(\cos x^2)(2x)}{(\sin(x^2) + 3)^2}$$

b) $y = \cot(\ln(x^2 + 2x))$

$$y' = -\csc^2(\ln(x^2 + 2x)) \left(\frac{1}{x^2 + 2x} \right) (2x + 2)$$

c) $y = \frac{\sec(x+2)}{\ln(\cos x)}$

$$y' = \frac{(\sec(x+2)\tan(x+2))(\ln(\cos x)) - \sec(x+2) \left(\frac{1}{\cos x} \right) (-\sin x)}{(\ln(\cos x))^2}$$

d) $y = (\cos(3x) - \cos^3 x)^4$

$$y' = 4(\cos(3x) - \cos^3 x)^3 (-\sin(3x) \cdot 3 + 3\cos^2 x \sin x)$$

$$e) y = \tan^3(e^{x^2-1}) \rightarrow (\tan(e^{x^2-1}))^3$$

$$y' = 3(\tan(e^{x^2-1}))^2 \cdot (\sec^2(e^{x^2-1})) \cdot (e^{x^2-1})(2x)$$

$$f) y = (3x - (3 - 2x^2)^3)^3$$

$$y' = 3(3x - (3 - 2x^2)^3)^2 \cdot (3 - 3(3 - 2x^2)^2) \cdot (-4x)$$

$$g) y = \frac{e^{x^2-2x}}{2x - \tan^{-1}x}$$

$$y' = \left[(e^{x^2-2x})(2x-2)(2x-\tan^{-1}x) - (e^{x^2-2x})(2 - \frac{1}{1+x^2}) \right] / (2x - \tan^{-1}x)^2$$

$$h) y = (x^2-x)\sin^{-1}(x^2)$$

$$y' = (2x-1)(\sin^{-1}(x^2)) + (x^2-x)\left(\frac{1}{\sqrt{1-(x^2)^2}}\right)(2x)$$

$$i) y = (\csc x)(\ln(1+\sin x))$$

$$y' = (-\csc x \cot x)(\ln(1+\sin x)) + (\csc x)\left(\frac{1}{1+\sin x}\right)(\cos x)$$

$$j) y = (x^3-x)\tan^{-1}(x)$$

$$y' = (3x^2-1)(\tan^{-1}(x)) + (x^3-x)\left(\frac{1}{1+x^2}\right)$$

$$k) y = \ln\left(\frac{(x+2)^3}{3+\cos x}\right) \rightarrow 3\ln(x+2) - \ln(3+\cos x)$$

$$y' = 3\left(\frac{1}{x+2}\right) - \left(\frac{1}{3+\cos x}\right)(-\sin x)$$

$$l) y = \tan \sin(\ln(2x^2-4x))$$

$$y' = \sec^2(\sin(\ln(2x^2-4x))) \cdot (\cos(\ln(2x^2-4x))) \cdot \left(\frac{1}{2x^2-4x}\right)(4x-4)$$

$$3. f(0) = 2; f'(0) = 3; g(x) = x + e^x; g'(x) = 1 + e^x \quad \downarrow \quad \downarrow$$

$$g(0) = 1, \quad g'(0) = 2$$

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$$(fg)'(0) = (3)(1) + (2)(2) = 3 + 4 = 7$$

$$\left(\frac{f}{g}\right)'(0) = \frac{(3)(1) - (2)(2)}{(1)^2} = \frac{3 - 4}{1} = -1$$

$$4. a) \lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}$$

$$\frac{\sin(5x)}{5x} \cdot \frac{1}{3} \cdot 5 = 1 \cdot \frac{1}{3} \cdot 5 = \frac{5}{3}$$

$$b) \lim_{x \rightarrow 0} \frac{x^2}{\sin(2x)\sin(3x)}$$

$$\left(\frac{2x}{\sin(2x)}\right) \left(\frac{3x}{\sin(3x)}\right) \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$5. f(x) = x e^{\sin x} \rightarrow (x)(e^{\sin x})$$

$$f'(x) = (e^{\sin x}) + (x)(e^{\sin x})(\cos x)$$

$$f''(x) = (e^{\sin x})(\cos x) + [(e^{\sin x})(\cos x) + (x)(e^{\sin x})(\cos^2 x) + (x)(e^{\sin x})(-\sin x)]$$

$$6. a) x \sec y = e^y \sin x$$

$$\sec y + x(\sec y \tan y)y' = (e^y)(y')(\sin x) + (e^y)(\cos x)$$

$$(x)(\sec y \tan y)y' - (e^y)(y')(\sin x) = (e^y)(\cos x) - \sec y$$

$$y' = \frac{e^y(\cos x) - \sec y}{x \sec y \tan y - e^y(\sin x)}$$

$$b) x^2 + y^2 = 2x^2 y^2$$

$$2x + 2yy' = 4xy^2 + 4x^2yy'$$

$$2yy' - 4x^2yy' = 4xy^2 - 2x$$

$$y' = \frac{4xy^2 - 2x}{2y - 4x^2y}$$

$$7. f(x) = (\tan x)^x \rightarrow y = (\tan x)^x$$

$$\ln y = x \ln(\tan x)$$

$$\frac{1}{y} \cdot y' = \ln(\tan x) + x \left(\frac{1}{\tan x} \right) (\sec^2 x)$$

$$y' = [\ln(\tan x) + x \left(\frac{1}{\tan x} \right) (\sec^2 x)] [(\tan x)^x]$$

Exam 3 MTH 229 Fall 2022 Total Pts:100 11/11/2022

Name: _____
Show all work for full credit.

Total Received:

1. A bacteria culture initially contains 50 cells and grows at a rate proportional to its size. After an hour the population has increased to 300. (10 Pts)
 - (a) Find an expression for the number of bacteria after t hours.
 - (b) Find the number of bacteria after 2 hours.
 - (c) Find the rate of growth after 2 hours.
 - (d) When will the population reach 5,000?
2. In a murder investigation, the temperature of the corpse was $32^\circ C$ at 5 PM and $30^\circ C$ an hour later. Normal body temperature is $37^\circ C$ and the temperature of the surroundings was $20^\circ C$. (10 Pts) (Hint: $T(t) = T_s + Ce^{kt}$, $T(0) = 32$)
 - (a) What is the temperature after 3 hours?
 - (b) When did the murder take place?
3. The length of a rectangle is increasing at a rate of 5 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 10 cm and the width is 12 cm, how fast is the area of the rectangle increasing? (8 Pts)
4. At noon, ship A is 500 mi west of ship B. Ship A is sailing south at 35 mi/hr and ship B is sailing north at 25 mi/hr. How fast is the distance between the ships changing at 3:00 P.M.? (10 Pts)
5. Find the linear approximation of the function $f(x) = \sqrt{x}$ at $a = 25$. Use it to approximate the number $\sqrt{26}$ and find the percentage error from this approximation. (8 pts)
6. Compare the values of Δy and dy if $y = f(x) = x^3 - 2x^2 + x + 1$ and x changes from 1 to 1.1. (6 Pts)
7. Find f if $f'(x) = x^2 + 2x - 3 \sin x$, $f(0) = 1$. (6 Pts)
8. First find f' and then find f for $f''(x) = 24x^2 - 6x + 3$, $f'(0) = 2$, $f(1) = 3$. (8)
9. Find the position of the particle if the particle is moving with the following data:
 $a(t) = \cos t - 2 \sin t$, $v(0) = 1$, $s(0) = 2$. (8 Pts)
10. Estimate the area under the graph of $f(x) = x^2 + 2$ from $x = 0$ to $x = 4$ using four rectangles and (i) right endpoints, (ii) left endpoints. (8 Pts)
11. Use the Fundamental Theorem of Calculus, Part I to find the derivative of the given function.
 - (a) $F(x) = \int_3^{e^x} \frac{t-2}{3t^2+4} dt$
 - (b) $G(x) = \int_{\ln x}^{\sin x} \frac{t^2-1}{\cos(t+1)} dt$ (6 Pts)
12. Evaluate the integral. (12 Pts)
 - (a) $\int_0^1 (3x^2 - e^x + \frac{1}{\sqrt{1-x^2}}) dx$
 - (b) $\int (\sec x \tan x + 3x^{\frac{3}{2}}) dx$
 - (c) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc \theta \cot \theta d\theta$
 - (d) $\int (3 \sec^2 x - 2 \tan x + \frac{1}{1+x^2}) dx$

$$1. B(0) = 50$$

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$$B(1) = 300$$

$$300 = 50e^k$$

$$\ln 6 = \ln e^k \quad k \approx 1.791759$$

$$a) B(t) = 50e^{1.791759t}$$

$$b) B(2) = 50e^{1.791759(2)}$$

$$B(2) = 1800$$

$$c) 1800 \times 1.791759 = 3225$$

$$d) 5000 = 50e^{1.791759t}$$

$$\ln 100 = \ln e^{1.791759t}$$

$$t = \frac{\ln 100}{1.791759} \approx 2.57 \text{ hrs}$$

$$2. T(0) = 32$$

$$T(1) = 30$$

$$T(t) = 20 + Ce^{kt} \rightarrow T(t) = 20 + 12e^{kt}$$

$$32 = 20 + C$$

$$C = 12$$

$$30 = 20 + 12e^k$$

$$\ln \frac{10}{12} = \ln e^k \quad k = -0.1823216$$

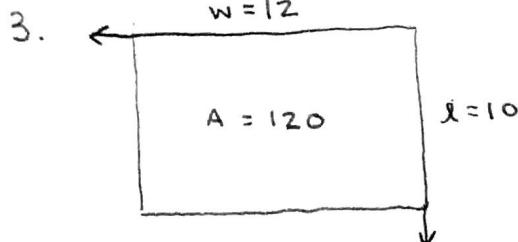
$$a) T(3) = 20 + 12e^{-0.1823216(3)}$$

$$T(3) = 26.94^\circ\text{C}$$

$$b) 37 = 20 + 12e^{-0.1823216t}$$

$$\ln \frac{17}{12} = \ln e^{-0.1823216t} \quad -0.191 \text{ hrs} = t$$

around 3 p.m.



$$\frac{dl}{dt} = 5 \text{ cm/s}$$

$$\frac{dw}{dt} = 3 \text{ cm/s}$$

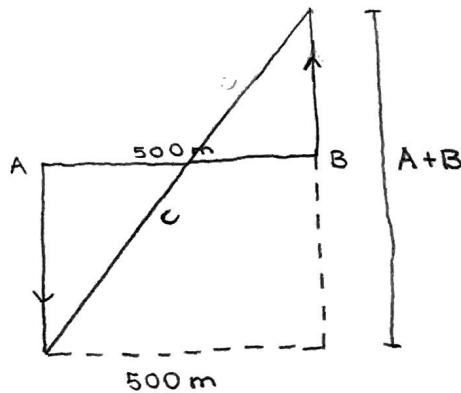
$$\frac{dA}{dt} = ?$$

$$A = lw$$

$$\frac{dA}{dt} = \frac{dl}{dt} \cdot w + \frac{dw}{dt} \cdot l$$

$$\frac{dA}{dt} = 5 \cdot 12 + 3 \cdot 10 = 90 \text{ cm/s}$$

4.



$$\frac{dA}{dt} = 35 \text{ mi/hr}$$

$$\frac{dB}{dt} = ? \quad * 3 \text{ hrs later}$$

$$\frac{dB}{dt} = 25 \text{ mi/hr}$$

$$\sqrt{(105 + 75)^2 + 500^2} = c \quad c \approx 531.4$$

$$(A+B)^2 + 500^2 = c^2$$

$$2(A+B) \left(\frac{dA}{dt} + \frac{dB}{dt} \right) = 2c \frac{dc}{dt}$$

$$2(105 + 75)(35 + 25) = 2(531.4) \frac{dc}{dt}$$

$$\frac{dc}{dt} = 20.32 \text{ mi/hr}$$

$$5. f(x) = \sqrt{x} = x^{1/2} \quad a=25$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(25) = \frac{1}{2 \cdot 5} = \frac{1}{10} = m$$

$$y = \frac{1}{10}x + b$$

$$5 = \frac{1}{10} \cdot 25 + b \quad b = 2.5$$

$$y = \frac{1}{10}x + 2.5 \approx \sqrt{x}$$

$$\frac{1}{10}(26) + 2.5 = 5.1 \rightarrow \frac{5.1 - 5.09902}{5.09902} \times 100\% = 0.0192\%$$

$$\text{actual: } \sqrt{26} \approx 5.09902$$

$$6. y = x^3 - 2x^2 + x + 1 \quad 1.1 - 1 = 0.1$$

$$dy = (3x^2 - 4x + 1) dx \leftarrow$$

$$= (3 - 4 + 1)(0.1) = 0$$

$$\Delta y = f(1.1) - f(1)$$

$$= [(1.1)^3 - 2(1.1)^2 + 1.1 + 1] - [1 - 2 + 1 + 1]$$

$$= 1.331 - 2.42 + 2.1 - 1$$

$$= 0.011$$

$$7. f'(x) = x^2 + 2x - 3 \sin x$$

$$f(x) = \frac{1}{3}x^3 + x^2 + 3 \cos x + C$$

$$1 = f(0) = \frac{1}{3}(0)^3 + (0)^2 + 3 \cos(0) + C$$

$$1 = 3 + C$$

$$f(x) = \frac{1}{3}x^3 + x^2 + 3 \cos x - 2$$

$$8. f''(x) = 24x^2 - 6x + 3$$

$$f'(x) = 8x^3 - 3x^2 + 3x + 2$$

$$f(x) = 2x^4 - x^3 + \frac{3}{2}x^2 + 2x - \frac{3}{2}$$

$$9. a(t) = \cos t - 2\sin t$$

$$v(t) = \sin t + 2\cos t - 1$$

$$s(t) = -\cos t + 2\sin t - t + 3$$

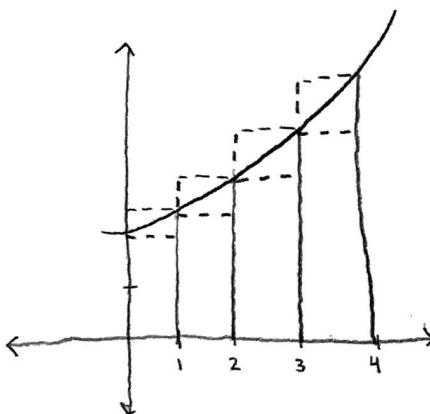
$$10. f(x) = x^2 + 2$$

$$\text{i)} R_4 = 1(3 + 6 + 11 + 18)$$

$$R_4 = 38$$

$$\text{ii)} L_4 = 1(2 + 3 + 6 + 11)$$

$$L_4 = 22$$



$$\text{II. a)} F(x) = \int_{-3}^{e^x} \frac{t-2}{3t^2+4} dt \rightarrow \frac{e^x - 2}{3e^{x^2} + 4} (e^x) = F'(x)$$

$$\text{b)} G(x) = \int_{\ln x}^{\sin x} \frac{t^2 - 1}{\cos(t+1)} dt$$

$$\rightarrow \frac{\sin^2 x - 1}{\cos(\sin x + 1)} (\cos x) - \left[\frac{2 \ln x - 1}{\cos(\ln x + 1)} \left(\frac{1}{x} \right) \right] = G'(x)$$

$$12. \text{ a) } \int_0^1 \left(3x^2 - e^x + \frac{1}{\sqrt{1-x^2}} \right) dx$$

$$x^3 - e^x + \sin^{-1}x \Big|_0^1 \rightarrow 1 - e^1 + \sin^{-1}1 - [0 - 1 + \sin^{-1}0] \\ \rightarrow 2 - e + \frac{\pi}{2}$$

$$\text{b) } \int (\sec x \tan x + 3x^{3/2}) dx$$

$$\sec x + \frac{6}{5} x^{5/2} + C$$

$$\text{c) } \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc x \cot x dx$$

$$-\csc x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -\csc \frac{\pi}{2} + \csc \frac{\pi}{4} = -1 + \sqrt{2}$$

$$\text{d) } \int (3\sec^2 x - 2\tan x + \frac{1}{1+x^2}) dx$$

$$3\tan x - 2 \ln |\sec x| + \tan^{-1} x + C$$

Final Exam MTH 229 Fall 2022 Total Pts:100 12/9/2022

Show all work for full credit. Write all your solutions on blank papers.

1. Use the method of substitution to evaluate the integral. (30 Pts)

(a) $\int \frac{e^x dx}{(e^x+1)^3}$

(b) $\int \frac{1}{x-3} dx$

(c) $\int x\sqrt{2x^2 - 3} dx$

(d) $\int (x+1) \sec^2(x^2 + 2x) dx$

(e) $\int_1^e \frac{(\ln x)^3}{x} dx$

(f) $\int \sin^4 x \cos x dx$

2. Evaluate the integral. (6 Pts)

(a) $\int_1^2 (4x^3 - \cos x + \frac{3}{x}) dx$ (b) $\int (\sec x \tan x + 2e^x - \frac{1}{1+x^2}) dx$

3. A farmer wants to fence an area of 450 square feet in a rectangular field and then divide it into three pens with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence? (8 Pts)

4. The top and bottom margins of a poster are each 5 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 320 cm^2 , find the dimensions of the poster with the smallest area. (8 Pts)

5. Summarize and then sketch the graph of a function that satisfies all of the conditions.

VAs: $x = -2, x = 2$, HA: $y = 0, f'(-4) = 0, f(0) = 0$

$f'(x) > 0$ if $x < -4$ or $x > 2$, $f'(x) < 0$ if $-4 < x < -2$ or $-2 < x < 2$

$f''(x) > 0$ if $-2 < x < 0, f''(x) < 0$ if $x < -2$ or $0 < x < 2$ if $x > 2$. (8 Pts)

6. For the function $f(x) = x^3 - 9x^2 + 15x - 2$, (10 Pts)

follow the following steps to sketch the curve without using calculator.

(a) Find the critical point(s).

(b) Find the intervals over which $f(x)$ is increasing and decreasing.

(c) Determine any local minimum and maximum values.

(d) Find the inflection point(s).

(e) Determine the intervals over which graph of $f(x)$ is concave up and concave down.

(f) Using all of the above, produce a sketch of the graph.

7. Find the absolute maximum value and absolute minimum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2,1]$. (6 Pts)

8. Use L'Hospital's Rule to find the following limits. (12 Pts)

(a) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$ (b) $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$ (c) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$

9. Use the guidelines to sketch the curve $f(x) = \frac{x-3}{x+2}$. (8 Pts)

$$1. (a) \int \frac{e^x}{(e^x + 1)^2} dx \quad u = e^x + 1 \\ du = e^x dx$$

$$= \int u^{-2} du \\ = -\frac{1}{u} + C = -\frac{1}{e^x + 1} + C$$

$$(b) \int \frac{1}{2x+3} dx \quad u = 2x+3 \\ du = 2dx$$

$$= \frac{1}{2} \int \frac{1}{u} du \\ = \frac{1}{2} \ln|2x+3| + C$$

$$(c) \int 5x \sqrt{x^2 - 3} dx \quad u = x^2 - 3 \\ du = 2x dx \\ \frac{1}{2} du = x dx$$

$$= \frac{5}{2} \left(\frac{u^{3/2}}{3/2} \right) + C = \frac{5}{3} \sqrt{(x^2 - 3)^3} + C$$

$$(d) \int (x+1) \csc^2(x^2 + 2x) dx \quad u = x^2 + 2x \\ du = (2x+2)dx \\ \frac{1}{2} du = (x+1)dx$$

$$= \frac{1}{2} (-\cot(x^2 + 2x)) + C$$

$$(e) \int_1^e \frac{(\ln x)^3}{x} dx \quad u = \ln x \\ du = \frac{1}{x} dx$$

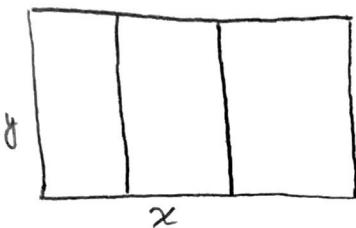
$$= \int_1^e u^3 du \\ = \left. \frac{(\ln x)^4}{4} \right|_1^e \Rightarrow \frac{1}{4}$$

$$(f) \int \sin^3 x \cos x dx \quad u = \sin x \\ du = \cos x dx \\ = \int u^3 du \\ = \frac{\sin^4 x}{4} + C$$

$$2. \int_1^2 (3x^2 - \cos x + \frac{2}{x}) dx \\ = x^3 - \sin x + 2 \ln|x| \Big|_1^2 \\ = (8 - \sin(2) + 2 \ln(2)) - (1 - \sin(1)) \\ = 7 - \sin(2) + 2 \ln(2) + \sin(1)$$

$$(b) \int (\sec x + \tan x + 2e^x - \frac{1}{1+x^2}) dx \\ = \sec x + 2e^x + \tan^{-1} x + C$$

3.



$$xy = 450 \text{ ft}^2$$

$$y = \frac{450}{x}$$

minimize perimeter: $2x + 4y$

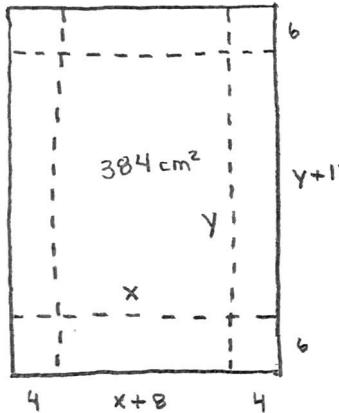
$$f(x) = 2x + 4\left(\frac{450}{x}\right) = 2x + \frac{1800}{x}$$

$$f'(x) = 2 - \frac{1800}{x^2} = 0$$

$$1800 = 2x^2 \Rightarrow x = 30 \text{ ft}, y = 15 \text{ ft}$$

$$\text{prove: } f''(30) = \frac{3600}{30^3} \leftarrow (+) \text{ minimum}$$

4.



$$xy = 384 \text{ cm}^2$$

$$y = \frac{384}{x}$$

minimize poster Area: $(x+8)(y+12)$

$$f(x) = (x+8)\left(\frac{384}{x} + 12\right) = 384 + 12x + \frac{3072}{x} + 96$$

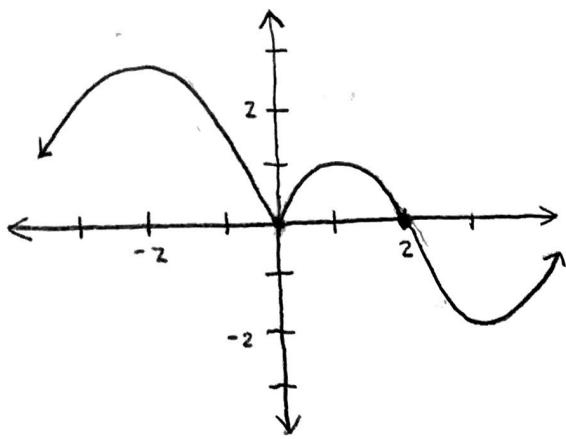
$$f'(x) = 12 - \frac{3072}{x^2} = 0$$

$$3072 = 12x^2 \Rightarrow x = 16, y = 24$$

$$\text{prove: } f''(16) = \frac{6144}{16^3} \leftarrow (+) \text{ minimum}$$

dimensions for poster are 24 cm x 36 cm

5.



6. $f(x) = 2x^3 - 3x^2 - 12$

(a) $f'(x) = 6x^2 - 6x = 6x(x-1) = 0 \Rightarrow \text{cr } \# : x=0, x=1$

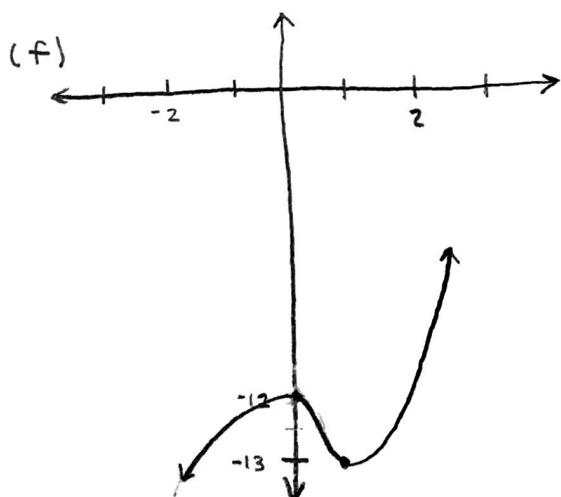
(b) $f'(x) :$

(c) local max : $(0, -12)$

local min : $(1, -13)$

(d) $f''(x) = 12x - 6 = 6(2x-1) = 0 \Rightarrow \text{IP} : x = \frac{1}{2}$

(e) $f''(x) :$



$$7. f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$$

$$\begin{aligned} f'(x) &= 6x^2 - 6x - 12 \\ &= 6(x^2 - x - 2) \Rightarrow \text{cr } \# : x = -1, 2 \end{aligned}$$

$$f(-2) = -3$$

$$f(-1) = 8 \leftarrow \text{absmax}$$

$$f(2) = -19 \leftarrow \text{absmin}$$

$$f(3) = -8$$

$$8. (a) \lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2} = \lim_{x \rightarrow 0} \frac{2e^{2x} - 2}{2x} = \lim_{x \rightarrow 0} \frac{4e^{2x}}{2} = 2$$

$$(b) \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{\sin x + x \cos x}{1 - \sin x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \frac{\cos x + \cos x - x \sin x}{-\sin x} = 2$$

$$(c) \lim_{x \rightarrow \infty} \frac{x^2 + 4x - 5}{e^x} = \lim_{x \rightarrow \infty} \frac{2x + 4}{e^x} = \lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$$

$$9. f(x) = \frac{x+3}{x-2} \quad x \neq 2 \quad y\text{-int: } (0, -\frac{3}{2})$$

$$f'(x) = \frac{(x-2) - (x+3)}{(x-2)^2} = \frac{-5}{(x-2)^2} \quad x\text{-int: } (-3, 0)$$

↓
no cr#

VA: $x=2$
HA: $y=1$

$$f'(x) = -5(x-2)^{-2} \quad f': \begin{array}{c} \nearrow \\ - \\ \frac{1}{2} \\ - \end{array}$$

$$f''(x) = \frac{10}{(x-2)^3}$$

$$f''(x): \begin{array}{c} \cap \\ CD \\ \leftarrow \end{array} \begin{array}{c} \cup \\ CU \\ \rightarrow \end{array} \begin{array}{c} (-) \\ 2 \\ (+) \end{array}$$

